5.9.2015
Contest

Contest Paper - May Mock AMC

Instructions

1. Do not open this booklet until your have set your timer to 75 minutes.
2. FORMAT: This contest contains 25 problems ranging from relatively straighforward to extremely advanced. Problems are roughly in order of increasing difficulty, although the range of difficulty is smaller than the actual AMC 10 and 12. Each problem is followed by 5 answer choices labelled A, B, C, D, and E. Only one answer is correct. You will have 1 hour and 15 minutes to work on these 25 problems.
3. Mark your answers on the AoPS Mock AMC google form, along with your AoPS username and your verification code that was private messaged to you.
4. SCORING: You may only give 1 answer for each problem. If you give more than 1 answer, all answers will be marked as incorrect. Each correct answer is worth 6 points; each unanswered problem will earn 1.5 points.
5. Only scratch paper, graph paper, rulers, protractors, and compasses are allowed as an aid in this contest. Calculators, dictionaires, geometric objects, and other aids are not permitted.
6. Figures may not be drawn to scale.
7. When finished, submit your google form. DO NOT discuss these problems or the solutions to these problems until May 11th, 2015.

1. Evaluate $(4*(-4)^2)((-2)^{-4})$
(A) -16 **(B)** 16 **(C)** 36 **(D)** 12 **(E)** 4

2. Anthony has 12 dollars after a shopping trip. During the shopping trip, he first spent $\frac{1}{2}$ of his money in the morning, then 5 dollars for lunch, then $\frac{2}{3}$ of his remaining money in the afternoon, and finally 2 dollars for the bus ride home. If he brought $\frac{1}{10}$ of his monthly salary on this shopping trip, how much is Anthony monthly salary?
(A) 812 **(B)** 376 **(C)** 1020 **(D)** 940 **(E)** 1024

3. If the average of n distinct positive integers is 7, and the smallest of these integers is 6, what is the maximum possible value of n?
(A) 1 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 7

4. What is the sum of the solutions of the quadratic $x^2 + 4x - 2016$?
(A) -4 **(B)** 4 **(C)** -20 **(D)** 504 **(E)** -504

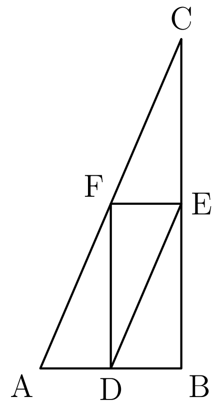
5. Define a new operation $\circ$ as $a \circ b = a^b + b$. If $9 \circ n = \frac{7}{2}$, what is the value of n?
(A) 2 **(B)** 3 **(C)** -2 **(D)** $\frac{1}{2}$ **(E)** $-\frac{1}{2}$

6. If we call an integer n "special" if we can write n as a sum of two distinct positive perfect cubes, how many special integers exist from 1 to 100, inclusive?
(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 9

7. A tarot deck is a deck of 78 cards consisting 4 suits, 21 picture cards, and a joker. The picture cards and the joker do not belong to any suit. Each suit contains the 13 normal cards ranks and an extra face card called a page (giving a total of 4 face cards per suit). What is the probability that Jack chooses 2 cards at random from a tarot deck (without replacement), such that Jack's cards are face cards of different suits?
(A) $\frac{8}{1001}$ **(B)** $\frac{16}{1001}$ **(C)** $\frac{32}{1001}$ **(D)** $\frac{64}{1001}$ **(E)** $\frac{48}{1001}$

8. A point $(0, 3)$ on the cartesian plane is rotated 90° clockwise about the origin, then reflected in the y-axis, rotated a half turn about the point $(2, 1)$, and finally reflected across the line $y = x$. What are the coordinates of the point after these transformations?
(A) $(0, 3)$ **(B)** $(1, 2)$ **(C)** $(3, 4)$ **(D)** $(2, 7)$ **(E)** $(1, 6)$

9. Triangle ABC has vertices $(0,0)$, $(6,0)$, $(6,14)$. The midpoints of all 3 sides of triangle ABC are connected to form a new triangle, DEF. Find the area within triangle ABC but outside triangle DEF.
(A) 30.75 **(B)** 31.25 **(C)** 31.5 **(D)** 31.75 **(E)** 32.25



10. If the linear equation $k(x-a)=0$ has a solution $x=3$, for some constant a with $a \neq 3$, how many possible values of k exist?
(A) 0 **(B)** 1 **(C)** 3 **(D)** Infinitely many **(E)** Cannot be determined from the information given

11. A prime number less than 20 is randomly chosen and cubed. Let this result be n. Find the probability that the remainder of the division $\frac{n}{3}$ is 2.
(A) $\frac{3}{8}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{7}{8}$

12. Let $\frac{2^{124}-1}{2^{62}-1}$ be x. Find the third-smallest prime factor of $4x-5$.
(A) 7 **(B)** 11 **(C)** 13 **(D)** 17 **(E)** 31

13. Let the real solutions of $(x^4-12x^3+35x^2)-(4x^2-48x+140)=0$ be a, b, c, and d. What is the value of $a+b+c+d$?
(A) 9 **(B)** 12 **(C)** 16 **(D)** 27 **(E)** 175

14. Right triangle ABC has $AB=\sqrt{15}$ and $AC=\sqrt{29}$. What is the length of BC?
(A) 4 **(B)** $\sqrt{44}$ **(C)** $\sqrt{57}$ **(D)** 8 **(E)** Cannot be determined from the information given

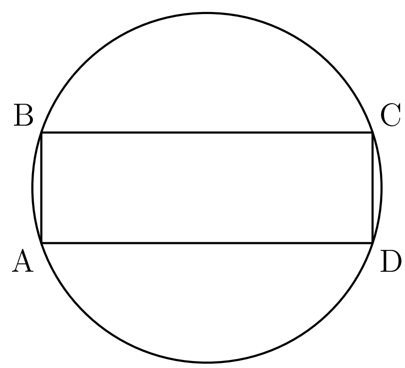
15. NOTE: This problem has been revised.
An originally un-painted 4x4x4 cube is painted red and then split into 8 2x2x2 cubes. Each of these are painted blue in any area without red paint. Finally, each of the 2x2x2 cubes are split into 8 1x1x1 cubes. How many of these cubes have red and blue on it?
(A) 32 **(B)** 48 **(C)** 56 **(D)** 60 **(E)** 64

16. Find the value of $\sqrt{\sqrt[3]{6 \cdot 24 \cdot 96} + 1}$.
(A) 1 **(B)** 3 **(C)** 5 **(D)** 9 **(E)** 25

17. If $2^5 3^4 4^3 5^9$ can be written in scientific notation as $x \cdot 10^y$, find the value of $\sqrt{100x} + y$.
(A) 6 **(B)** 10 **(C)** 14 **(D)** 21 **(E)** 29

18. How many ways can 8 candies be distributed to 4 kids such that each kid gets at least 1 candy, and Ally and Brian, 2 of the 4 kids, get the same amount of candies?
(A) 1 **(B)** 4 **(C)** 9 **(D)** 16 **(E)** 25

19. A rectangle $ABCD$ has area 42 and perimeter 34. What is the diameter of the circle that passes through A, B, C, and D?



(A) $\sqrt{205}$ **(B)** 15 **(C)** $\sqrt{221}$ **(D)** 16 **(E)** 17

20. Coin A is tossed 7 times, and coin B is tossed 6 times. What is the probability that more heads are tossed using coin A than using coin B?
(A) $\frac{1}{3}$ **(B)** $\frac{6}{13}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{17}{32}$

21. At 9:00 AM, Austin starts riding his bike north down a highway at 20 kilometres per hour. 1 hour later, Jessica starts driving her car south down the same highway at 60 kilometres per hour. Austin's starting point and Jessica's starting point are 150 kilometres apart, with Austin on the north end. At 10:30 AM, a train moving north passes Austin. At 11:15 AM, that same train passes Jessica. If the train maintains a constant speed, what is that speed (in kilometres per hour)?
(A) 60 **(B)** 70 **(C)** 90 **(D)** 100 **(E)** 120

22. Find the value of
$(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2015}+\frac{1}{2016})(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2014}+\frac{1}{2015})-(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{2015}+\frac{1}{2016})(\frac{1}{2}+\frac{1}{3}...\frac{1}{2014}+\frac{1}{2015})$
(A) $\frac{1}{2013}$ **(B)** $\frac{1}{2014}$ **(C)** $\frac{1}{2015}$ **(D)** $\frac{1}{2016}$ **(E)** $\frac{1}{2017}$

23. A hexagon $ABCDEF$ is constructed, along with diagonals AD, BE, CF. Alan the ant starts on point A. He crawls around the hexagon, along the segments we drew. After crawling along 6 points, he stops. How many points from the set $S = A, B, C, D, E, F$ can Alan end up?
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

24. Find the sum of all values of x that satisfy $(x^2-4x+2)^{x^2-5x+2}=1$.
(A) -7 **(B)** 1 **(C)** 6 **(D)** 13 **(E)** 16

25. In an ulitmate frisbee tournament, x countries participate. Each country sends n teams, team 1 through team n, to the tournament. During the tournament, each team is to play each other team from a different country twice. How many games are in the tournament in terms of x and n?
(A) $2n^2(x^2-x)$ **(B)** $n^2(x^2-x)$ **(C)** n^2x^2 **(D)** $\frac{2nx}{(x-1)^2}$ **(E)** $x^2(n^2+n)$