Name: Grade:

2015 BCA Mock AMC 10

- There are 25 questions to be answered over a period of 75 minutes. This mimics the format of the AMC 10/12.
- Each question has 5 answer choices, exactly one of which is correct.
- You will receive 6 points for every correct answer and 1.5 points for every question you leave blank. Thus blind guessing is not in your favor. If you are able to eliminate 2 or more answer choices, it is generally favorable for you to guess.

ANSWERS (circle at most one per question):

1	A	B	C	D	E	6	A	B	C	D	E	11	A	B	C	D	E
2	A	B	C	D	E	7	A	B	C	D	E	12	A	B	C	D	E
3	A	B	C	D	E	8	A	B	C	D	E	13	A	B	C	D	E
4	A	B	C	D	E	9	A	B	C	D	E	14	A	B	C	D	E
5	A	B	C	D	E	10	A	B	C	D	E	15	A	B	C	D	E

16	A	B	C	D	E	21	A	B	C	D	E
17	A	B	C	D	E	22	A	B	C	D	E
18	A	B	C	D	E	23	A	B	C	D	E
19	A	B	C	D	E	24	A	B	C	D	E
20	A	B	C	D	E	25	A	B	C	D	E

NAME: GRADE:

1. Kelvin the Frog can butter 3 pieces of bread in 4 minutes and can pour 6 drinks in 2.5 minutes. How long does it take him to prepare a meal for 12 people if all of them want a drink but only half of them want a piece of bread?

   (A) 9 (B) 13 (C) 15.5 (D) 18 (E) 21

2. An obtuse, isosceles triangle has one angle measuring 20 degrees. What is another angle in this triangle?

   (A) 40 (B) 80 (C) 120 (D) 140 (E) 160

3. Kelvin the Frog has 32 nickels and Alex the Kat has 2 quarters. Kelvin gives Alex a certain amount of his nickels so they have the same amount of money. How many coins does Alex now have?

   (A) 11 (B) 12 (C) 13 (D) 14 (E) 15

4. Kelvin the Frog loves flies. In the deepest part of the jungle, there are blue flies, green flies, gooey flies, and mean flies. How many flies must Kelvin eat to guarantee that he eats at least two of the same type of fly?

   (A) 2 (B) 4 (C) 5 (D) 8 (E) 9

5. AJ the Dennis is at the center of a 2 ft by 2 ft square. He runs to one vertex and then to another vertex, both times in a straight line. What is the maximum distance he could have traveled?

   (A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) $2+\sqrt{2}$ (E) $3\sqrt{2}$

6. Kelvin the Frog must wait in the jungle until Alex the Kat rolls two dice and ends up with a sum of 5 or a sum of 8. If Alex the Kat rolls both dice exactly once, what is the probability that Kelvin can leave the jungle?

   (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

7. In the land of dystopia, people use a different currency than in America. In dystopia, 7 *Despair*'s equals 15 *Orwell*'s and 3 *Huxley*'s equals 5 *Despair*'s. Which of the following is greatest?

   (A) 3 *Despair*'s (B) 3 *Huxley*'s (C) 3 *Orwell*'s (D) 2 *Huxley*'s and 1 *Orwell* (E) 2 *Despair*'s and 1 *Huxley*

8. The math team is putting together a 2-person team for a competition. 8 members will only do Algebra, 5 members will only do Geometry, and 2 members are willing to do either. If the 2-person team must include one person doing Algebra and one person doing Geometry, how many different teams can be formed?

   (A) 42 (B) 43 (C) 56 (D) 66 (E) 67

9. There are 10 members on a math team, but only 8 of them compete in any given competition. Given any two members of the team, there was exactly one competition that they both did not compete in. How many competitions did each math team member compete in?

   (A) 27 (B) 28 (C) 36 (D) 45 (E) 55

10. Arturo is learning not-spanish and realizes that the not-spanish alphabet is the same as the english alphabet, with two extra letters: ñ and ll. If he picks two distinct letters from the not-spanish alphabet, what's the probability exactly one of them is also an english letter?

    (A) $\frac{1}{14}$ (B) $\frac{13}{189}$ (C) $\frac{26}{189}$ (D) $\frac{52}{189}$ (E) $\frac{13}{14}$

11. Mr. Roboto takes a number, squares each of its digits, and adds the resulting numbers. So if the Styx handed Mr. Roboto the number 23, Mr. Roboto would give them 13, because $13 = 2^2 + 3^2$. One day, Mr. Roboto malfunctions and, instead of handing back the number he's supposed to, repeats the process 2014 more times (for a total of 2015 operations). If the Styx handed Mr. Roboto 292, what number would they be handed back?

    (A) 37 (B) 42 (C) 58 (D) 89 (E) 292

12. Points E and F lie inside quadrilateral $ABCD$ such that $\angle DAE = \angle EAF = \angle FAB$ and $\angle ADE = \angle EDF = \angle FDC$. If $\angle ABC = 120°$ and $\angle BCD = 90°$, find $\angle AED + \angle AFD$ in degrees.

    (A) 150 (B) 180 (C) 210 (D) 240 (E) 270

13. Kelvin the Frog's favorite number, n, is a perfect square when 42 is added to it and a perfect fourth power when 1337 is added to it. Compute the sum of the digits of n^2.

    (A) 12 (B) 14 (C) 16 (D) 18 (E) 20

14. Let $P(x)$ be a quadratic polynomial satisfying $P(1) = 1, P(2) = 8, P(3) = 27$. Find $P(4)$.

    (A) 46 (B) 54 (C) 58 (D) 64 (E) 81

15. Let ABC be a triangle such that $AB = 5$, and $CA = 6$. Let D be the intersection of the angle bisector of $\angle A$ with side BC and M be the midpoint of side BC. Given that $BD = 3$, compute DM.

(A) 0.3 (B) 0.4 (C) 0.5 (D) 0.6 (E) 0.7

16. A sphere with radius r_1 is inscribed in a cone with volume 54. A smaller sphere with radius r_2 lies on top of the original sphere such that it lies tangent to the cone and the bottom sphere. If the volume of the smallest cone containing the smaller sphere is 2, find $\frac{r_2}{r_1}$.

(A) $\frac{1}{27}$ (B) $\frac{1}{9}$ (C) $\frac{1}{3\sqrt{3}}$ (D) $\frac{1}{3}$ (E) $\frac{1}{\sqrt{3}}$

17. Kelvin the Frog starts at the upper left dot in a 6x6 grid of dots. Kelvin wishes to reach the opposite corner through a series of hops, each either down or to the right. However, Ryan the Ryan will eat Kelvin if Kelvin hops to any of the central 4 dots. How many different paths can Kelvin take without being eaten?

(A) 34 (B) 52 (C) 70 (D) 132 (E) 252

18. The roots of the quadratic $x^2 + ax + b$ are r and s, and the roots of the quadratic $x^2 + bx + a$ are $r + 1$ and $s + 1$. Find ab.

(A) -3 (B) -1 (C) 0 (D) 1 (E) 3

19. What is the last digit of $1^{2015} + 2^{2015} + \ldots + 2015^{2015}$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

20. The number 2015 has the property that each of the first 3 digits are less than the last digit. How many 4-digit numbers, including 2015, have this property?

(A) 285 (B) 1092 (C) 1337 (D) 1740 (E) 2024

21. Kelvin the Frog is solving a 2015×2015 crossword puzzle that has exactly one black square in every column. Kelvin is stuck and decides to cheat by moving the black squares. Every minute, he can move any one of the black squares either up or down 1 square. Let n be the expected number of minutes it will take Kelvin to move all of the squares to the central row. Find the sum of the digits of n.

(A) 17 (B) 18 (C) 19 (D) 24 (E) 27

22. In a certain fantasy game, all characters begin with a positive integer k armor, which blocks $\frac{k}{k+10}$ of an incoming attack. For example, 40 armor would block 80% of an incoming attack. Kelvin the Frog notices that, after buying some integer amount of additional armor, he now blocks 10% more damage than he would before his purchase. What is the largest possible amount of armor that Kelvin could have bought? For example, Kelvin might have started with 40 armor and bought an additional 50, thus going from 80% damage reduction to 90% damage reduction.

(A) 5 (B) 50 (C) 810 (D) 9801 (E) 9890

23. Kelvin the Frog and Alex the Kat stand at two distinct vertices of an equilateral triangle, the sides of which are perfect mirrors with length 1. Kelvin the Frog shoots a laser at some angle $0 < \theta < 60°$ (i.e. within the interior of the triangle), and hits Alex the Kat after some finite amount of bounces. What is the length of the shortest path that the laser could have traveled?

(A) $\sqrt{3}$ (B) $\sqrt{5}$ (C) $\sqrt{6}$ (D) $\sqrt{7}$ (E) $2\sqrt{3}$

24. We say $a \equiv b \pmod{m}$ if and only if $m \mid a - b$. Suppose a, b, c are positive integers less than 13 such that

$$\begin{aligned} 2ab + bc + ca &\equiv abc \pmod{13} \\ ab + 2bc + ca &\equiv 3abc \pmod{13} \\ ab + bc + 2ca &\equiv 5abc \pmod{13} \end{aligned}$$

Find the remainder when $a + b + c$ is divided by 13.

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

25. Define $f(b, x)$ to be the number of terminal zeros there are when x is written in base b; i.e. the number of zeros x ends in when written in base b. For example, $f(6, 360) = 2$ as $360 = 1400_6$. Find the number of $2 \leq b \leq 100$ such that $f(b, 100!)$ is odd.

(A) 43 (B) 44 (C) 45 (D) 48 (E) 54