2015 Mock AMC 10

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1 Contest Rules

Do NOT proceed to the next page until you have read all of the rules and your timer has started.

1. This is a twenty-five question multiple choice test. Each question is followed by the answers A, B, C, D, and E. Only one of these is correct.

2. Mark your answers on the Google Form given in the AoPS thread. Submit with your AoPS username and a method of verification.

3. You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.

4. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators are allowed. No problems on the test will require the use of a calculator.

5. Figures are not necessarily drawn to scale.

6. You will be given 75 minutes to complete the test. Please do not spend longer than this, or it will hurt the integrity of the contest.

Good luck!

"Always finish what you start."

$\sim$ droidguyBOY

Thanks to AkshajK, DrMath, and Not_A_Username for test solving!
(oops AkshajK why did you leave and go to MOP before giving us comments)
Special thanks to DrMath for helping us finalize before release!

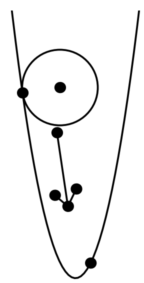
2 Problems

1. What is the value of $(2-0+1)^5-(20-1+5)$?

(A) 2015 **(B)** 219 **(C)** -23 **(D)** 8 **(E)** -1771561

2. A clock of diameter 10 sits tangent to the parabola $f(x)=\frac{x^2}{2}-3x+1$ at the point $(-4, f(-4))$ such that it is above the parabola. If the clock remains tangent to the parabola and rolls around, with its center's x-coordinate increasing, until the clock is tangent at the point $(5, f(5))$, what is the area of the clock?

(A) 16π **(B)** 100π **(C)** 9π **(D)** 25π **(E)** 81π



3. Franklyn's favorite number gives the same remainder when divided by 6, 19 or 42. If his number is greater than 4000, what is the smallest possible value of his favorite number?

(A) 4778 **(B)** 4788 **(C)** 4798 **(D)** 4808 **(E)** 4818

4. For how many integer values x is x^4-11x^2+10 negative?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

5. Alex, Betty, Cathy, and Daniel are participating in a footrace. After months of hard work, all four of them are hoping to achieve first place in the race. In how many different orders can the four of them finish if ties are allowed?

(A) 15 **(B)** 30 **(C)** 60 **(D)** 75 **(E)** 150

6. Find the last digit of 42^{1337}.

(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

7. Vaibhav is eating some dumplings. He has 3 chicken dumplings, 2 shrimp dumplings, and 921030129 apple dumplings. As Vaibhav is feeling especially fat today, he decides to randomly choose 7 dumplings to eat. In how many different ways can Vaibhav choose these 7 dumplings, given that he considers dumplings of the same type as identical?

(A) 8 **(B)** 10 **(C)** 12 **(D)** 16 **(E)** 18

8. A hexagonal prism has the same volume as a square pyramid. If the side length of the base of the prism is the same as the side length of the base of the pyramid, what is ratio of the height of the taller polyhedron to the height of the shorter polyhedron?

(A) $\frac{15}{4}$ **(B)** $5\sqrt{2}$ **(C)** $\frac{10\sqrt{3}}{3}$ **(D)** $\frac{9\sqrt{3}}{2}$ **(E)** $4\sqrt{3}$

9. What is the sum of all the digits D that will make the number 4D4D45 divisible by 15?

(A) 12 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 18

10. Triangle ABC has $\angle A = 15°, \angle B = 120°, AC = 8$. Compute the area of $\triangle ABC$.

(A) $16 - \dfrac{16\sqrt{3}}{3}$ **(B)** 16 **(C)** $\dfrac{16\sqrt{3}}{3}$ **(D)** $16 + \dfrac{16\sqrt{3}}{3}$ **(E)** 7

11. Compute the sum of all integers n such that the quantities $\dfrac{2n}{n+8}$ and one more than five halves of the aforementioned quantity are equal.

(A) -4 **(B)** -2 **(C)** 0 **(D)** 2 **(E)** 4

12. Given a and b are real numbers such that $a + b = 6$ and $a^2 + b^2 = 24$, find $a^4 + b^4 - 4a^2b^2$.

(A) 300 **(B)** 324 **(C)** 360 **(D)** 432 **(E)** 504

13. Kaan has a very strange disease, and must visit Doctor Math's three offices regularly. He likes the WOOT(832) office the most, so he visits once every 3 days. He likes the WOOT(971) office too, so he visits once every 8 days. However, he does not like the downtown office, as it is full of janitors and MOPers, so he only visits once every 2 weeks. Yesterday, he went to all three offices. What is the number of days in the next 365 days in which Kaan will visit exactly two of the offices?

(A) 17 **(B)** 20 **(C)** 23 **(D)** 26 **(E)** 29

14. Nathan Hu and Ben Qi are playing a game called "count-down". When given a signal to start, the first player to slap their buzzer wins. Someone always wins, as the sensor is very pro. If Ben is faster 90% of the time, and they play 6 games, what is the probability that they have won the same number of games at the end?

(A) $\dfrac{81}{50000}$ **(B)** $\dfrac{729}{100000}$ **(C)** $\dfrac{729}{1000000}$ **(D)** $\dfrac{81}{1000000}$ **(E)** $\dfrac{729}{50000}$

15. Bhavy the fatty starts at the origin of a coordinate plane. Given that he is on (a, b), he moves according to the following set of rules:

I. If a and b are both even, he will move to $(a + 3, b + 3)$
II. If a and b are both odd, he will move to $(a + 2, b + 3)$
III. If a and b are neither both even, nor both odd, he will move to $(a + 1, b + 2)$

What is the positive difference between the coordinates of the point Bhavy will be on after 2015 moves?

(A) 1337 **(B)** 1340 **(C)** 1343 **(D)** 2013 **(E)** 2015

16. What is the largest possible radius of a semicircle that can be drawn in an isosceles trapezoid with parallel bases of lengths 10 and 20, and legs of length 13?

(A) 7 **(B)** $\frac{110}{13}$ **(C)** $\frac{120}{13}$ **(D)** 10 **(E)** $\frac{150}{13}$

17. An equilateral triangle in the cartesian plane has one of its sides lying on the line $y = 2x - 4$. If one of its vertices lies on the origin, what is the perimeter of the triangle?

(A) $\frac{8\sqrt{15}}{5}$ **(B)** $\frac{7\sqrt{3}}{2}$ **(C)** $\frac{5\sqrt{6}}{2}$ **(D)** $2\sqrt{15}$ **(E)** $\frac{11\sqrt{3}}{3}$

18. Ryan the Turtle decides to go on a walk. While he is walking, he sees a fair coin with a heads side and a tails side. He picks it up and decides to flip it 6 times. What is the probability that tails never occur on consecutive flips?

(A) $\frac{5}{16}$ **(B)** $\frac{21}{64}$ **(C)** $\frac{11}{32}$ **(D)** $\frac{23}{64}$ **(E)** $\frac{3}{8}$

19. If the roots of polynomial $2x^3 - 4x^2 + 7x - 5$ are a, b, c, find the value of

$$\frac{1}{(b-4)(a-2)+2a-4} + \frac{1}{(b-4)(c-2)+2c-4} + \frac{1}{(a-4)(c-2)+2c-4}.$$

(A) $\dfrac{5}{3}$ **(B)** $\dfrac{4}{5}$ **(C)** $\dfrac{7}{5}$ **(D)** $\dfrac{-5}{3}$ **(E)** $\dfrac{8}{9}$

20. Two distinct two digit prime numbers less than 60 are chosen. When their sum is subtracted from their product, which of the following values is impossible to attain?

(A) 1137 **(B)** 1287 **(C)** 1439 **(D)** 1559 **(E)** 1623

21. $\triangle ABC$ has $AB = 13$, $BC = 14$, and $AC = 15$. The internal angle bisector of A intersects BC at D. A perpendicular from C intersects the extension of $\overline{AD}$ at E. Find CE.

(A) $\dfrac{12\sqrt{65}}{13}$ **(B)** $\dfrac{96}{13}$ **(C)** $\dfrac{2\sqrt{365}}{5}$ **(D)** $\dfrac{3\sqrt{365}}{2}$ **(E)** $\dfrac{13\sqrt{365}}{14}$

22. AjitBakshajKadaverboy rides the bus home from Shortfellow Middle School, while the great AkshajBakshajKakshaj rides the bus home from TJHS for History and English. If Ajit arrives home at a random time between 3 : 00 and 5 : 00 PM and stays awake for 50 minutes before going to bed, while Akshaj arrives home at a random time between 4 : 00 and 6 : 00 PM and stays awake for 70 minutes before going to bed, what is the probability that Ajit and Akshaj are awake together for more than 10 minutes and less than 30 minutes?

(A) $\dfrac{1}{4}$ **(B)** $\dfrac{5}{36}$ **(C)** $\dfrac{1}{6}$ **(D)** $\dfrac{1}{8}$ **(E)** $\dfrac{1}{12}$

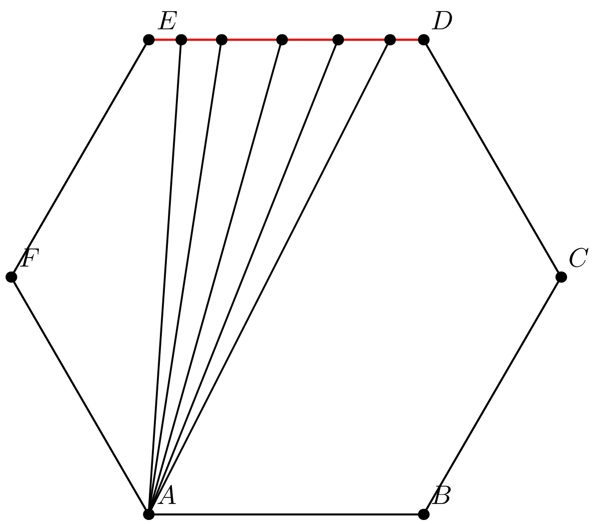
23. Sum the series

$$\frac{1}{4}+\frac{1}{4}+\frac{9}{64}+\frac{1}{16}+\frac{25}{1024}+\dots$$

(A) $\frac{2}{3}$ **(B)** $\frac{19}{27}$ **(C)** $\frac{20}{27}$ **(D)** $\frac{7}{9}$ **(E)** $\frac{22}{27}$

24. Kevin Liu of the Indiana Army and Daniel Zhu of the Married Land are playing "Not-Enders Game" in in a regular hexagonal room $ABCDEF$ with walls of length 4 that reflect rays shot from stun-guns. Daniel is sitting at corner E two corners down from Kevin at corner A. If Kevin wants to paralyze Daniel with his stun-gun, but must aim his first shot at a point on the wall ED that is neither E or D, due to there being a picture of 2015 JMO 2 on that wall, what is length of the shortest path the stun-gun ray can take?

(A) $8\sqrt{10}$ **(B)** $4\sqrt{39}$ **(C)** 25 **(D)** $10\sqrt{6}$ **(E)** $4\sqrt{41}$



25. Andy wants to find the roots of the polynomial $(1+x+x^2)(2+4x-10x^2+5x^3+x^4)-1-17x^3$. While solving, he finds that one real, non-integer root can be expressed in the form $\frac{n+\sqrt{z}+\sqrt{g}}{y}$, where n, g, y are expressions and z is a positive integer. He then finds that $g = a - b\sqrt{c}$ for some positive integer a, b, c with c squarefree. Compute c.

(A) 17 **(B)** 19 **(C)** 21 **(D)** 22 **(E)** 23

3 Tiebreaker Round

TIEBREAKER 1:

Whoever submits first wins tiebreaks! :P