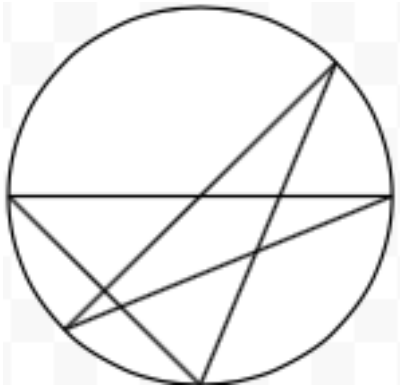
Mock AMC 10/12

This is a test that you have 75 minutes to take. You may use scratch paper, a pencil, a protractor, a compass, and a ruler. No other math tools are permitted such as a calculator. Submit your answers to joey8189681 listing the question number and then your answer A-E. If there is a grammatical error in here, please don't pm me unless it directly affects the question itself. Like "they're" instead of "their." I highly doubt that there will be one but still…. Qualifying scores will make the Mock AIME. This will be taken later. So without further ado, here it is.

1. Josh is riding his bike up a hill at an average rate of 8 mi/h. If the hill is 10 miles long, how many minutes will it take Josh to get to the end of the hill?
(A) 30 (B) 45 (C) 48 (D) 60 (E) 75

2. Bill receives \$20/hr. for his job. Jenny makes \$35/hr. in her job. Bill and Jenny agree to split the cost of 2 tickets to the Beethoven concert in a week. Each ticket to the concert costs \$350. Given that neither of them has any money right now, how many more hours per day must Bill work than Jenny in order to buy a ticket.
(A) $\frac{5}{28}$ (B) $\frac{1}{2}$ (C) $\frac{15}{14}$ (D) $\frac{15}{4}$ (E) $\frac{15}{2}$

3. The figure below is a star inscribed in a circle. The 5 inscribed angles make up the stars "vertex interior angles." Find the sum of the degree measure of all the "vertex interior angles."
(A) 180 (B) 270 (C) 360 (D) 450 (E) 540



4. Pranav's mom walks to college at a rate of 5 mph. She then leaves college at a rate of 3 mph. If she travels the same distance walking to college as leaving college, what was Pranav's mom average speed?
(A) $\frac{7}{2}$ (B) $\frac{15}{4}$ (C) 4 (D) $\frac{17}{4}$ (E) 4π

5. The number 6 is called 5-guarded because the sum of the two squares it is in between are 5 (2 and 3). Find the value of n, if 2015 is n-guarded.
(A) 87 (B) 89 (C) 91 (D) 93 (E) 95

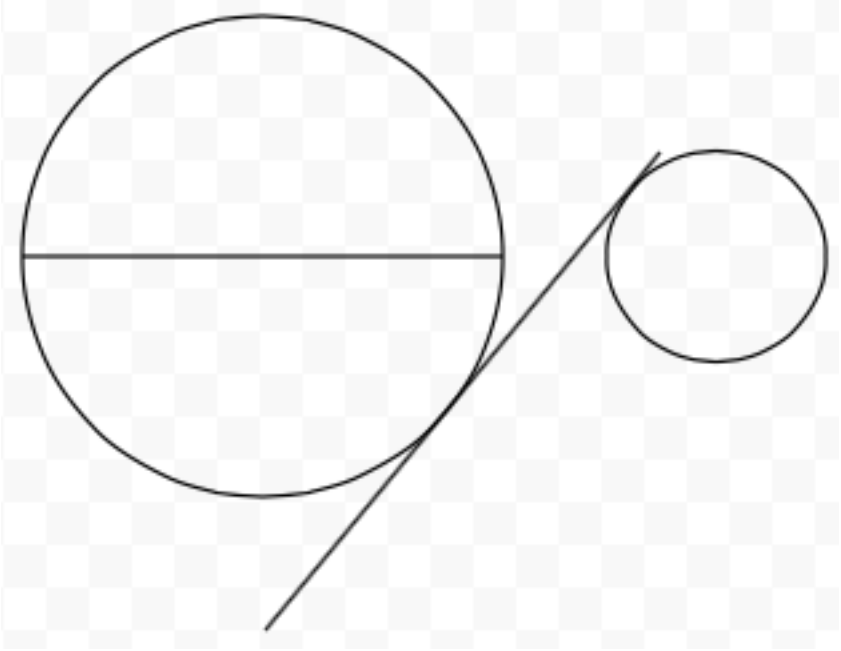
6. A 25 by 25 square has the same center as a 12 by 15 rectangle. What is the area inside the square but outside the rectangle?
(A) 400 (B) 425 (C) 445 (D) 461 (E) 481

7. An arithmetic and geometric sequence share the same three first terms in the same order. What is the sum of the common ratio of the geometric sequence and the common difference of the arithmetic sequence?
(A) $-\frac{11}{4}$ (B) 0 (C) 1 (D) $\frac{11}{4}$ (E) 3

8. What is the maximum number of points of intersection between two circles and two lines?
(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

9. Jenny flips a fair coin 4 times in a row. What is the probability that she flips either exactly two heads in a row or exactly two tails in a row, but not both?
(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) $\frac{5}{8}$

10. As shown in the diagram below, two circles are externally tangent to a common line. The ratio of the radius of the larger circle to the smaller circle is 3. Also, the ratio of the length of the larger circles center to the smaller circle's point of tangency to the length of the smaller circles center to the larger circle's point of tangency is 2. What is the ratio of the smaller circles radius to the distance between the points of tangency?
(A) $\frac{1}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{15}}{5}$ (D) $\frac{4}{5}$ (E) $\frac{\sqrt{3}}{2}$



11. For a given quadratic function $f(x)$, $f(1) = 3, f(2) = 9$, and $f(3) = 27$. What is the value of $f(7)$?
(A) 219 (B) 625 (C) 1324 (D) 1850 (E) 2187

12. A triangle is bounded by the lines, $y = a, x = b$, and $y = cx$, where a, b, and c, are all positive, real numbers. Find the area of the triangle.
(A) $|a^2 - b^2|$ (B) $|a^2 + b^2 - 2c^2|$ (C) $\left|\frac{a^2}{2c} + \frac{b^2c}{2} - ab\right|$
(D) $\left|\frac{a^2+b^2}{2c} - ab\right|$ (E) $\left|\frac{b^2}{c} - abc + \frac{a^2c}{2} + \frac{b^2c}{2}\right|$

13. A baker can serve 5 different types of bread. A family of 5 visiting the baker is going to sit down at a round table in such a way that no two people next to each other have the same type of bread. In how many ways can the baker put down the bread if the people will choose which type of bread they want when they get to the table (assuming they are not there yet), and the person in seat 4 gets Rye bread?
(A) 192 (B) 204 (C) 240 (D) 256 (E) 288

14. Evaluate $\sqrt{7+2\sqrt{3}}+\sqrt{7-2\sqrt{3}}-\sqrt{14-2\sqrt{37}}$.

(A) $2\sqrt{3}$ (B) $2\sqrt{7-2\sqrt{3}}$ (C) $2\sqrt{14-2\sqrt{37}}$ (D) 4 (E) $3\sqrt{2}$

15. What is the value of the sum,

$$\frac{1}{3}+\frac{2}{9}+\frac{1}{9}+\frac{4}{81}+\frac{5}{243}+\cdots$$

(A) $\frac{1}{2}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

16. Joey was feeling very sympathetic for making a few hard problems in the middle of the contest. Then he debated over whether or not to make the next one easy. He rolled a pair of dice to determine how difficult he would make the next couple of problems. What is the probability that if he rolls the pair of dice twice, that the total product of all four of the dice has exactly 12 positive factors?

(A) $\frac{1}{9}$ (B) $\frac{4}{27}$ (C) $\frac{14}{81}$ (D) $\frac{16}{81}$ (E) $\frac{2}{9}$

17. Jean and Tracy want to go to an amusement park. They both go sometime between 9AM and 4PM and stay for an additional 5 hours. If they are both at the amusement park at the same time, they have a $\frac{1}{5}$ chance of seeing each other. What is probability that they will see each other at the amusement park?

(A) $\frac{19}{144}$ (B) $\frac{5}{36}$ (C) $\frac{1}{7}$ (D) $\frac{1}{6}$ (E) $\frac{9}{49}$

18. A clock has an hour hand and a minute hand. The hour and minutes hands are extended to be tangent to a smaller circle that is tangent to the clock. The ratio of the radius of the clock to the radius of the circle is $\sqrt{6}+\sqrt{2}-1$. Which of the following could be the time?

(A) 1:00 (B) 1:11 (C) 1:15 (D) 1:26 (E) 2:00

19. An equiangular hexagon has side lengths of 6, 8, 12, 6, 8, and 12 in that order. What is the area of this hexagon?
(A) $96\sqrt{3}$ (B) $108\sqrt{3}$ (C) 192 (D) $144\sqrt{3}$ (E) $192\sqrt{3}$

20. What is the hundreds digit of 2015^{1337}?
(A) 1 (B) 3 (C) 5 (D) 7 (E) 9

21. In America, 2% of people get fatism, a disease where your mass is too much to count. There is a machine that will tell you whether or not a person will contract fatism. But, the machine only has a 99% accuracy rate. Of all of the people who get a reading that says they will contract fatism, what fraction of them actually will contract it?
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{99}{148}$ (D) $\frac{100}{149}$ (E) $\frac{7}{10}$

22. How many times does the graph of $f(x) = 20\sin(.5x)$ intersect the graph of $f(x) = \frac{x}{2}$? (Measured in radians)
(A) 5 (B) 11 (C) 12 (D) 13 (E) 15

23. How many integers from 50-59, inclusive, can be the hypotenuse of a right triangle with integer side lengths?
(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

24. A quadrilateral ABCE is inscribed in circle O. In the quadrilateral, $AE = 8, AB = 5, BC = 6,$ and $CE = 8$. Given both AC and EB are integers; the area of $ABCE$ can be expressed in the form $\frac{a\sqrt{b}}{c} + d\sqrt{e}$, where gcd (a, c)=1, and b and e are not divisible by any perfect squares other than 1. What is the value of $a + b + c + d + e$?
(A) 25 (B) 125 (C) 225 (D) 325 (E) 425

25. Two weighted coins are flipped, each of a different weight. One coin will give heads every time. The non-constant coin has an equal probability of flipping 2 heads in a row to flipping 2 tails and then 1 head in that order. The two coins are placed into a box and are chosen randomly, with replacement. After repeating a process of choosing and then flipping, 1 tail has been flipped and 4 heads have been flipped. The probability that the constant coin was drawn more than the non-constant coin is $\frac{k+m\sqrt{n}}{p}$ where n is not divisible by the square of any prime and the fraction is as simple as possible. What is the value of $k + m + n + p$?

(A) 37 (B) 56 (C) 81 (D) 104 (E) 144