2015 'Hard' Mock AMC 8

Contest Rules

Do NOT proceed to the next page until you have read all of the rules.

1. This is a twenty-five question multiple choice test. Every question will be followed by the answer choices A, B, C, D, and E. Only one of these will be correct.

2. Mark your answers on the Google Form given in the AoPS thread. Submit with your AoPS username and the verification code we PMed you with.

3. No aids are permitted other than scratch paper, writing utensils, and erasers. No problems on the test will require the use of a calculator. Computers are only allowed for viewing the questions and submitting.

4. Diagrams are not necessarily drawn to scale.

5. Every question is worth one point. There is no penalty for guessing.

6. You will have 40 minutes to complete this test. Please do not spend longer than this to keep the integrity of the contest.

Good luck and have fun!

1. Compute $1^2 + 2^2 - 3^2 + 4^2 + 5^2 - 6^2 + 7^2 + 8^2 - 9^2 + 10^2$.

   (A) 19   **(B)** 89   **(C)** 133   **(D)** 259   **(E)** 385

2. Below is a 4x4 puzzle with each row, column, and quadrant with the digits 1-4 each used once with no repetitions. What number should replace the question mark?

   (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** Not Enough Information

3. If $f(x) = x^2 - x - 7$, What is $f(f(f(3)))$?

   (A) -5 **(B)** -1 **(C)** 6   **(D)** 23   **(E)** 36

4. Michael and Nancy each drew a circle. Michael says: My circle has a circumference of 36π. Nancy says: My circle has an area of 36π! Assuming that they are both telling the truth, what is the ratio of the radius of Nancy's circle to Michael's circle?

   (A) $\dfrac{1}{6}$ **(B)** $\dfrac{1}{4}$ **(C)** $\dfrac{1}{3}$ **(D)** $\dfrac{1}{2}$ **(E)** $\dfrac{2}{3}$

5. Andy sells apples. He sold $\frac{2}{3}$ thirds of them to Billy. Then, Cameron came and bought $\frac{1}{2}$ of the remaining apples, leaving Andy with only 4 left. How many apples did Billy buy?

   (A) 8   **(B)** 16   **(C)** 24   **(D)** 32   **(E)** 48

6. Ben has a box of donuts, and he has more than three donuts. When he tries to put his donuts in groups of 4, 15, or 24, there are 3 donuts left over. What is the least possible number of donuts in the box?

   (A) 60   **(B)** 63   **(C)** 120   **(D)** 123   **(E)** 1443

7. Jane and her sister are both under the age of 50. Jane is 1 year younger than half of her sister's age. Nine years ago both of them had an age that was a perfect cube. How old is Jane's sister?

   (A) 8   **(B)** 10   **(C)** 17   **(D)** 27   **(E)** 36

8. What is the sum of the prime factors of $6^4 + 6^2 + 1$?

   (A) 42   **(B)** 48   **(C)** 60   **(D)** 68   **(E)** 74

9. There are 4 people: A, B, C, and D. Out of them 4, there is only 1 truth teller. They make the following statements below:

$$A : \text{I tell the truth}$$

$$B : \text{Person A is lying, only I tell the truth}$$

$$C : \text{Person B is lying}$$

$$D : \text{Only I tell the truth}$$

Who could be the truth teller?

(A) A only   (B) B only   (C) A or D   (D) B or C   (E) None

10. Two regular polygons, A and B, have an equal perimeter. Polygon A has an interior angle of 165 degrees. Polygon B has an exterior angle of 10 degrees. What is the ratio of the length of a side of polygon A to a side of polygon B?

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{2}{3}$ (D) $\dfrac{3}{2}$ (E) $\dfrac{5}{2}$

11. What is the sum of the possible values of $x + y$ in the following equation:

$$\frac{x(2y + x) + y^2}{7} + 1 - y = x$$

(A) -7 (B) -2 (C) 2 (D) 424 (E) 7

12. When Jack wakes up everyday, he picks a sock. There is 1 blue sock, 2 white socks, 3 red socks, and 4 yellow socks. Because Jack is biased in his pick, the probability of him picking out a blue sock is 4 times as likely as a yellow sock, 3 times as likely as a red sock, and 2 times as likely as a white sock. What is the probability of him picking a yellow or white sock?

(A) $\dfrac{1}{5}$ (B) $\dfrac{9}{25}$ (C) $\dfrac{3}{5}$ (D) $\dfrac{16}{25}$ (E) $\dfrac{5}{8}$

13. Isosceles Triangle ABC is shown below. Points D and E are the midpoints of line AB and AC respectively. Line DE has length 16. The line from point A to the midpoint of line BC has length 30. What is the ratio of line DF, to line FC?
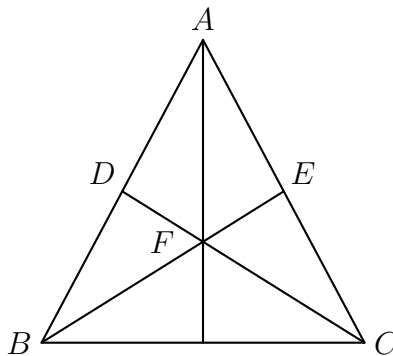
(A) $\dfrac{8}{23}$ (B) $\dfrac{4}{9}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{8}{15}$ (E) $\dfrac{3}{4}$

14. How many positive even factors does 2016 have?

    (**A**) 8   (**B**) 10   (**C**) 15   (**D**) 30   (**E**) 60

15. The sum of the lengths of all the diagonals of a unit cube can be expressed in the form $a\sqrt{b} + c\sqrt{d}$, where all roots are simplified to lowest terms. Find $a + b + c + d$. Space and face diagonals should be included in your answer.

    (**A**) 18   (**B**) 21   (**C**) 24   (**D**) 27   (**E**) 30

16. Four friends each have a different pokemon card. They put their pokemon cards into a pile of four, and then each person randomly chooses one pokemon card. In how many ways can they do this, such that at least one person ends up with their own pokemon card?

    (**A**) 1   (**B**) 9   (**C**) 12   (**D**) 15   (**E**) 23

17. In acute $\triangle ABC$, draw AD such that D is on BC. Draw DX such that X is on AC and DX is perpendicular to AC. If $m\angle XDB = 2m\angle CAD + 6$, and $m\angle ADB = 2m\angle XDC - 12$, what is $m\angle ACB$?

    (**A**) 24   (**B**) 36   (**C**) 48   (**D**) 60   (**E**) 72

18. A cylinder of diameter of 16 in. and a height of 20 in. is drilled $\frac{3}{5}$ of the way through the center of the base to the center of the other base. The hole happens to be a cone with diameter of 10 in. as shown below. Find the surface area of the resulting figure.

    (**A**) 488π (**B**) 512π (**C**) 548π (**D**) 640π (**E**) 720π
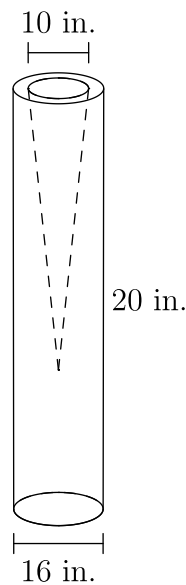
19. How many positive integer factors of 2015^2 are less than 2015?

    (**A**) 7   (**B**) 8   (**C**) 13   (**D**) 14   (**E**) 17

20. Define a_n to be $x^n + \frac{1}{x^n}$. If $a_1 = 3$, what is the sum of the digits of a_{11}?

    (**A**) 17   (**B**) 18   (**C**) 19   (**D**) 20   (**E**) 21

21. A number is called perfect digital if the sum of its digits is a perfect square. How many 3 digit perfect digital numbers are there?

    (**A**) 126   (**B**) 128   (**C**) 165   (**D**) 189   (**E**) 212

22. Right triangle ABC is inscribed in a circle of area C where $100 < C < 200$. The side lengths of the triangle are of length y, x, and $x + 1$, where x and y are integers. What is the radius of the largest circle that can be inscribed in the medial triangle of triangle ABC? Express your answer in simplest form. (The medial triangle is defined to be the triangle formed by connecting the midpoints of a triangle.)

    (**A**) $\dfrac{1}{2}$ (**B**) $\dfrac{2}{3}$ (**C**) 1   (**D**) $\dfrac{3}{2}$ (**E**) 2

23. During a flight on an airplane, Eric strikes up a chat with Erica, the person sitting next to him. It turns out that Erica has two kids, and at least one of them is a girl born on a Tuesday. Being a mathematician, Eric decides to find the probability that Erica has a boy and a girl before asking her. What is the probability that Erica has one boy, and one girl? Assume an equal chance of giving birth to either gender and an equal chance to giving birth on any day.

    (**A**) $\dfrac{1}{3}$ (**B**) $\dfrac{7}{20}$ (**C**) $\dfrac{1}{2}$ (**D**) $\dfrac{14}{27}$ (**E**) $\dfrac{2}{3}$

24. Forrest and Alex have 9 distinct marbles. To divide them between each other, Forrest will first take a group of marbles (he can take all of them, or none of them). Alex will then choose one marble from the remaining marbles, add it to Forrest's group of marbles, and take the rest of the marbles. In how many ways can this be done?

    (**A**) 81   (**B**) 512   (**C**) 1800   (**D**) 2304   (**E**) 2700

25. Compute the remainder when $5^{30} \cdot 3^{93}$ is divided by 67.

    (**A**) 2   (**B**) 9   (**C**) 27   (**D**) 45   (**E**) 66

Problem Authors

Question 1: NaU

Question 2: 8Invalid8

Question 3: 8Invalid8

Question 4: 8Invalid8

Question 5: 8Invalid8

Question 6: 8Invalid8

Question 7: 8Invalid8

Question 8: NaU

Question 9: 8Invalid8

Question 10: 8Invalid8

Question 11: 8Invalid8

Question 12: NaU

Question 13: 8Invalid8

Question 14: NaU

Question 15: 8Invalid8

Question 16: NaU

Question 17: NaU

Question 18: 8Invalid8

Question 19: NaU

Question 20: NaU

Question 21: 8Invalid8

Question 22: 8Invalid8

Question 23: NaU

Question 24: NaU

Question 25: NaU