2017 Mock AMC 8

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Instructions

1. DO NOT BEGIN THIS TEST UNTIL YOU HAVE SET A TIMER FOR 40 MINUTES.
2. This is a 25-question multiple choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Submit your answers for each problem on the 2017 MOCK AMC 8 through a private message to eisirrational. DO NOT EDIT THE FIRST POST OF YOUR PM.
4. SCORING: You will receive 1 point for each correct answer, and 0 points for each incorrect answer.
5. When submitting your answers, only submit exactly 25 letters; do not submit the numerical answer. If you do so, your submission will not be graded. (We say exactly 25 because even if you cannot solve a problem, it is better to guess than not submit anything - there is no penalty for guessing.)
6. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
7. Figures are not necessarily drawn to scale.
8. After you turn the page, begin working on the problems. You will have **40 minutes** to complete the test.
9. Enjoy the problems and good luck!

Other information such as errata and where to submit will be located in eisirrational's post.

The Problems

1. How many '9's are in the number $1001^2 - 2 \cdot 1001$?
(A) 0 **(B)** 2 **(C)** 4 **(D)** 6 **(E)** 8

2. The first day that Jaime buys his favorite chocolate cake, he eats one-fourth of the cake. Each day after that, Jaime eats 25% less cake than the day before. At the end of the third day, what fraction of the cake remains?
(A) $\frac{1}{4}$ **(B)** $\frac{21}{64}$ **(C)** $\frac{27}{64}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{1}{2}$

3. Two positive integers have a greatest common factor of 2 and a least common multiple of 30. What is the largest possible sum of the two numbers?
(A) 16 **(B)** 21 **(C)** 25 **(D)** 32 **(E)** 36

4. Barnabus has 4 cards that are labelled on the front and the back. One has 1 on the front and 2 on the back, one has 2 and 3, one has 4 and 5, and the last one has 6 and 7. He wants to lay the cards out in a way so that the numbers facing up add up to 15. What is the number of ways to do this, if the order of the cards does not matter?
(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

5. A teacher has graded all of the tests besides one. So far, the students have gotten the scores $40, 72, 74, 75, 78, 80, 82, 84, 86, 98, 100$. After the teacher grades the last test, the average score is 79. What was the last student's score?
(A) 73 **(B)** 75 **(C)** 77 **(D)** 79 **(E)** 81

6. In circle O, points A, B, and C are on O so that O is in the exterior of triangle ABC. Line OC intersects AB in the interior of the circle at P. If $\angle BAC = 20$ and $\overset{\frown}{AC} = 90$, find $\angle BPC$.
(A) 45 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 65

7. In triangular pyramid $ABCD$, points E, F, G are location on AB, AC, AD so that $\frac{AE}{AB} = \frac{AF}{AC} = \frac{AG}{AD} = \frac{17}{20}$. Also let H is the intersection of altitudes of triangle BCD. If the ratio of the volumes of $EFGH$ and $ABCD$ is $\frac{a}{b}$, find $b - a$.
(A) 391 **(B)** 2220 **(C)** 3087 **(D)** 7133 **(E)** 7973

8. Al, Bob, and Carl make the following statements:
Al: Both Bob and Carl are liars.
Bob: At least one of Al and Carl are liars.
Carl: Exactly one of Al and Bob are liars.

   Who is/are the liar(s)?
(A) Al **(B)** Bob **(C)** Carl **(D)** Al and Carl **(E)** Bob and Carl

9. The sum of all possible distinct permutations of a positive integer N, whose digits are all nonzero, is 1332. What is the largest possible product of digits of N?
(A) 18 **(B)** 30 **(C)** 42 **(D)** 54 **(E)** 66

10. How many positive integers less than 1000 have a digit sum of 12? Some numbers to include are 39 and 525.
(A) 55 **(B)** 66 **(C)** 73 **(D)** 79 **(E)** 80

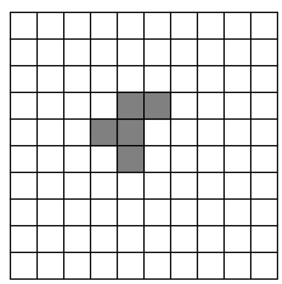
11. What is the smallest value of n so that $2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}$ with n 2's is greater than $4^{4^{\cdot^{\cdot^{\cdot^{4}}}}}$ with 10 4's?
(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

12. An urn has ten marbles, five of which are red, three of which are white, and two of which are blue. Sam selects four marbles such that there is at least one marble of each color in his set. How many ways can Sam select his set of four marbles?
(A) 60 **(B)** 75 **(C)** 90 **(D)** 100 **(E)** 105

13. A group of people went to a restaurant. However, as they were preparing to pay, one person objected to paying because she did not eat anything. Thus, everybody had to pay 2 extra dollars per person. Just before paying, another person objected because he did not eat anything either. Thus, everybody had to pay an additional 3 dollars. How many dollars did the meal cost?
(A) 60 **(B)** 75 **(C)** 90 **(D)** 96 **(E)** 120

14. In a basket lies 169 marbles, some of them red and the rest blue, of which there are more blue marbles than red marbles. If Katia randomly picks two marbles, the probability that they are of the same color is the same as the probability that they are of different colors. How many red marbles were there?
(A) 52 **(B)** 65 **(C)** 78 **(D)** 84 **(E)** 85

15. Fabio's favorite pentomino (tile with five squares) is the F-pentomino, shaded below (the shape can be rotated or reflected). How many ways can Fabio shade five squares of a 10×10 grid such that the selected squares form an F-pentomino?



(A) 256 **(B)** 300 **(C)** 400 **(D)** 512 **(E)** 600

16. Which of the following is equal to $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdot^{\cdot^{\cdot}}}}}$?
(A) $\sqrt[4]{2}$ **(B)** $\sqrt{2}$ **(C)** 1 **(D)** 2 **(E)** 4

17. A triangle has lattice point coordinates, with no two x-coordinates the same and no two y-coordinates the same. What is the minimum possible area of the triangle? (A lattice point is a point (a, b) in the plane, where a and b are integers.)
(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** 1 **(E)** $\frac{3}{2}$

18. Among the first 101 positive perfect squares, one is chosen at random. What is the probability that the chosen number leaves a remainder of 1 when divided by 12?
(A) $\frac{7}{101}$ **(B)** $\frac{13}{101}$ **(C)** $\frac{17}{101}$ **(D)** $\frac{26}{101}$ **(E)** $\frac{34}{101}$

19. Let n be the number of ways that the letters of the word MATHEMATICS can be rearranged to form a different eleven-letter "word" so that no two vowels are adjacent. Find the number of factors of n.
(A) 60 **(B)** 84 **(C)** 108 **(D)**162 **(E)** 216

20. We define a *partition* of a positive integer n to be a set of positive integers that sum to n. Define a partition to be *unique* if each positive integer in the set is distinct. Let $P(N)$ be the maximum number of integers in the group of a unique partition of N. For how many positive integers $n \leq 2017$ does

$P(N) + 1 = P(N + 1)$?
(A) 61 **(B)** 62 **(C)** 63 **(D)** 64 **(E)** 65

21. Let ABC be a triangle with $\angle A = 42^\circ$. The largest circle that will fit inside ABC is drawn with center I. Label D, E, F as the intersection between AI, BI, CI with the circle, respectively. Tangent lines to the circle at D, E, F are drawn so that they form another triangle XYZ, where X, Y, Z are opposite A, B, C respectively. What is the measure of $\angle X$?
(A) 42° (B) 45° (C) 48° (D) 60° (E) 69°

22. Al, Alex, Alexa, Alexander, and Alexandra all start with \$100. Whenever one person gives money to someone else, they give 10% of their money to that person - this encounter is called an *exchange*. Among the five people, five exchanges occur, where each person gives money to someone else at least once. After this process, the maximum amount of money that Al can end up with is closest to which of the following answer choices?
(A) 100 **(B)** 110 **(C)** 120 **(D)** 130 **(E)** 140

23. Let ABC be an acute triangle so that $AB = \sqrt{61}$ and $AC = \sqrt{37}$. A square $PQRS$ has P, Q on BC. Lines QR, RS intersect AC at the trisection points of AC, and PS, RS intersect AB at the trisection points of AB. Find BC.
(A) $\frac{4}{\sqrt{5}}$ **(B)** $\sqrt{\frac{132}{7}}$ **(C)** $\frac{10}{\sqrt{3}}$ **(D)** $\frac{2\sqrt{70}}{5}$ **(E)** 6

24. In a bin are 37 strings, of length $1, 2, 3, \cdots, 37$. Odd-length strings are red, and even-length strings are blue. The probability of picking a string is proportional to its length. If two strings are drawn simultaneously, the probability that one of them is red and the other is blue is $\frac{a}{b}$ where a and b are relatively prime. What is a?
(A) 19 **(B)** 2166 **(C)** 361 **(D)** 1881 **(E)** 4181

25. Consider equilateral triangle ABC. We draw circle O such that it is tangent to sides $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ and let P be the point of tangency between circle O and side $\overline{CA}$. There exists a point D on side $\overline{BC}$ such that $\overline{AD}$ bisects $\overline{OP}$. Given that $OP = 7$, what is the value of CD?
(A) $\frac{7}{2}\sqrt{3}$ **(B)** $28\sqrt{3} - 42$ **(C)** $4\sqrt{3}$ **(D)** 7 **(E)** $5\sqrt{3}$