MAA

MATHEMATICAL ASSOCIATION OF AMERICA

MAA American Mathematics Competitions

32nd Annual

AMC 8

American Mathematics Contest 8

Tuesday, November 15, 2016

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOUR PROCTOR TELLS YOU.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the AMC 8 Answer Form with a #2 pencil. Check the blackened circles for accuracy and erase errors and stray marks completely. Only answers properly marked on the answer form will be graded.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your proctor will ask you to record your information on the answer form.
8. You will have 40 minutes to complete the test once your proctor tells you to begin.
9. When you finish the exam, *sign your name* in the space provided on the answer form.

The Committee on the American Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it determines that the required security procedures were not followed.

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1. What is $\dfrac{2^0 \cdot (1-8)}{20 \cdot 1 + 8}$?

   (A) 0 **(B)** $-\dfrac{1}{4}$ **(C)** $\dfrac{8}{29}$ **(D)** $\dfrac{1}{4}$ **(E)** undefined

2. Aaron buys a couch for \$225 and sells the same couch for \$240. Erin buys a phone for \$690 and sells it for \$750. What percent of Aaron's amount of profit is Erin's amount of profit?

   (A) 12.5 **(B)** 25 **(C)** 112.5 **(D)** 125 **(E)** 400

3. Find the least prime number that is one more than a fourth power.

   (A) 2 **(B)** 5 **(C)** 7 **(D)** 17 **(E)** 257

4. Four people are chosen for a math competition. How many ways are there to have someone get the highest score and have someone get the lowest score? There are no ties.

   (A) 4 **(B)** 6 **(C)** 12 **(D)** 16 **(E)** 24

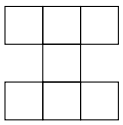
5. Find the sum of the solutions to the equation $x^2 - 24x + 144 = 0$. Roots with a multiplicity of more than 1 only count towards the sum once.

   (A) -12 **(B)** 12 **(C)** 24 **(D)** 25 **(E)** 144

6. Let $a \star b = a^2 + b^3 + 5$. What is the value of $(1 \star 5) + ((6 \star 2) \star 3)$?

   (A) 2400 **(B)** 2564 **(C)** 2712 **(D)** 2776 **(E)** 3416

7. Seven squares are arranged in a certain way, shown below. What is the perimeter of the resulting figure if its area is 175?



(A) 60 **(B)** 80 **(C)** 100 **(D)** 120 **(E)** 140

8. How many solutions exist to the equation $\dfrac{|x-2|(x-3)(x-7)}{x^2-10x+21} = 5$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

9. What is the sum, in degrees, of all exterior angles of a dodecagon (12-gon)?

(A) 15 **(B)** 30 **(C)** 180 **(D)** 360 **(E)** 1800

10. Alvin, Simon, and Theodore all took the same test. Simon's score was three times Alvin's score, and Theodore's score was five less than half of Simon's score. If the sum of all their scores was 171, what was Theodore's score?

(A) 19 **(B)** 32 **(C)** 43 **(D)** 71 **(E)** 96

11. Find the area of a right triangle if its hypotenuse is 12 and one leg is 7.

(A) $\sqrt{95}$ **(B)** $7\sqrt{95}$ **(C)** $\frac{7}{2}\sqrt{95}$ **(D)** $\frac{7}{4}\sqrt{95}$ **(E)** 31.5

12. What is the probability of flipping at least 2 tails if you flip a fair coin 5 times? It only lands on heads or tails.

(A) $\dfrac{1}{2}$ **(B)** $\dfrac{5}{8}$ **(C)** $\dfrac{11}{16}$ **(D)** $\dfrac{3}{4}$ **(E)** $\dfrac{13}{16}$

13. How many ordered pairs of positive coprime integers a and b are there such that a and b are both less than 10? Two integers are considered coprime if their greatest common factor is 1.

(A) 45 **(B)** 55 **(C)** 63 **(D)** 69 **(E)** 75

14. Find the value of $\dfrac{x^9 \times x^{20} \times 200}{x^{25}}$ if $x = 3$.

(A) 5400 **(B)** 8100 **(C)** 12600 **(D)** 16200 **(E)** 21600

15. Find the perimeter of a right triangle with with integer side lengths and the shortest side 8.

(A) 24 **(B)** 32 **(C)** 40 **(D)** 48 **(E)** 56

16. The number $2018AB2018$ is divisible by 3, where $B > A$. How many ordered pairs (A, B) are possible?

(A) 15 **(B)** 18 **(C)** 24 **(D)** 30 **(E)** 36

17. On an island, there are only knights, who always tell the truth, and knaves, who always lie. 300 islanders stand in a circle. Let N be the maximum number of knights if all islanders in the circle say they are adjacent to exactly one knight. Let n be the maximum number of knights if all islanders in the circle say they are adjacent to no knights. What is $N - n$?

(A) 0 **(B)** 25 **(C)** 50 **(D)** 75 **(E)** 100

18. The mean and range of ten consecutive terms in an arithmetic progression are all 82. What is the first term in the sequence?

(A) 41 **(B)** 52 **(C)** 67 **(D)** 72 **(E)** 77

19. Find the sum of the remainders when 6^{2018} is divided by each of 1, 2, 3, 4, 5, 6, 7, 8, and 9.

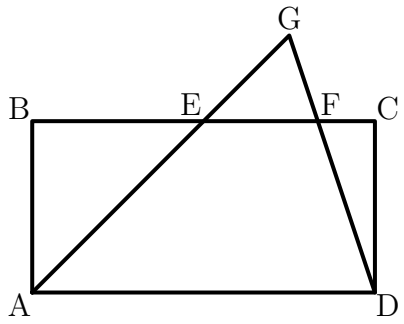
(A) 1 **(B)** 2 **(C)** 3 **(D)** 8 **(E)** 12

20. How many different times, ignoring seconds, are there on an analog clock in 24 hours when the hour and the minute hands are separated by an angle equal to that swept by the minute hand in 2 minutes?

(A) 46 **(B)** 48 **(C)** 50 **(D)** 52 **(E)** 54

21. Points E and F lie on segment BC of rectangle $ABCD$. Segments AE and DF are extended such that they intersect at a point G outside of the rectangle, shown below. Given that AB and BE are both 6 units long, AD is 12 units long, and FC is 2 units long, what is the area of triangle $\triangle EFG$, in square units?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 12



22. In $\triangle ABC$, $AB = 3, BC = 4, CA = 5$. Line L is parallel to the angle bisector of $\angle ABC$ such that it hits $\overline{CA}$ at its midpoint. L is extended so that it hits $\overline{AB}$ at point P. Find the area of $\triangle APC$.

(A) $\frac{13}{2}$ **(B)** 7 **(C)** $\frac{15}{2}$ **(D)** 8 **(E)** $\frac{17}{2}$

23. A classroom has at least one boy and one girl, and no more than 40 students in total. Two students are chosen to give class presentations at random, without replacement. If the probability that they are both boys is half the probability that one is a boy and one is a girl, and the probability that they are both girls is non-zero, what is the sum of all possible class sizes?

(A) 384 **(B)** 388 **(C)** 392 **(D)** 396 **(E)** 400

24. How many ways are there to seat three indistinguishable kids in a row of 11 chairs if nobody can sit next to one another?

(A) 84 **(B)** 86 **(C)** 88 **(D)** 120 **(E)** 504

25. If a, b, c, and k are real numbers such that $a+b+c = 5$, $a^2+b^2+c = 15$, and $ab = k$, what is the smallest possible value of k?

(A) -5 **(B)** $\frac{-41}{8}$ **(C)** $\frac{-21}{4}$ **(D)** $\frac{-43}{8}$ **(E)** $\frac{-11}{2}$