Christmas Mock AMC 8

mathchampion1

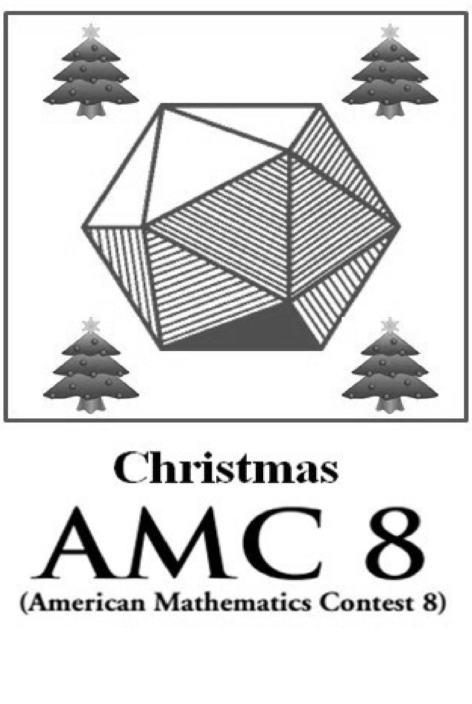
December 2018

1 Introduction

Merry Christmas; thank you for interest in this mock!

The standard rules of an AMC 8 apply.
The bubble answer sheet is on the next page, and test begins on the page after.

Good luck and have fun!



Name ______________________________

Christmas AMC 8 Answer Sheet

1. (A) (B) (C) (D) (E)
2. (A) (B) (C) (D) (E)
3. (A) (B) (C) (D) (E)
4. (A) (B) (C) (D) (E)
5. (A) (B) (C) (D) (E)
6. (A) (B) (C) (D) (E)
7. (A) (B) (C) (D) (E)
8. (A) (B) (C) (D) (E)
9. (A) (B) (C) (D) (E)
10. (A) (B) (C) (D) (E)
11. (A) (B) (C) (D) (E)
12. (A) (B) (C) (D) (E)
13. (A) (B) (C) (D) (E)
14. (A) (B) (C) (D) (E)
15. (A) (B) (C) (D) (E)
16. (A) (B) (C) (D) (E)
17. (A) (B) (C) (D) (E)
18. (A) (B) (C) (D) (E)
19. (A) (B) (C) (D) (E)
20. (A) (B) (C) (D) (E)
21. (A) (B) (C) (D) (E)
22. (A) (B) (C) (D) (E)
23. (A) (B) (C) (D) (E)
24. (A) (B) (C) (D) (E)
25. (A) (B) (C) (D) (E)

2 Problems

1. What is two less than three times the answer to this problem?

(A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

2. If the ratio of one number to another is $11:15$, then what is the ratio of that same number to the average of the two numbers?

(A) $11:26$ **(B)** $6:13$ **(C)** $1:2$ **(D)** $7:13$ **(E)** $11:13$

3. Which of these positive integers shares no common positive divisors with any of the others?

(A) 48 **(B)** 57 **(C)** 77 **(D)** 102 **(E)** 134

4. What is the greatest integer less than or equal to $\dfrac{1\times 2\times 3+3\frac{1}{2}+4\times(5-6)\times 7}{8-9+10-11}$?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

5. If $\log_a c = b$ is another way to write the relation $a^b = c$, then which of the following quantities is smallest in numerical value? (The answer choices for this problem are not necessarily ordered from least to greatest.)

(A) 0 **(B)** $\log_{2018} 2$ **(C)** $\sqrt[2018]{2}$ **(D)** $\sqrt[2018^{2018}]{2018}$ **(E)** $\log_2 \dfrac{1}{2018}$

6. How many sum totals can be rolled with 10 standard six-sided dice? Assume that all dice must be used in any given sum total.

(A) 45 **(B)** 50 **(C)** 51 **(D)** 60 **(E)** 71

7. Three sides of a quadrilateral have lengths 12, 15, and 30. Which of the following is a possible length of the fourth side?

(A) $\sqrt{5}$ **(B)** $\sqrt{7}$ **(C)** 36 **(D)** 58 **(E)** 81

8. Based on the reports of a shoe company, it is a generally observed fact that foot radius varies in direct proportionality with shoe size. A particular shoe of size 10 corresponds to a 5-inch foot radius. If the shoe's size is increased to 11, then by how much must the foot radius be increased for the shoe to still fit?

(A) $\frac{1}{8}$ in. **(B)** $\frac{1}{4}$ in. **(C)** $\frac{1}{2}$ in. **(D)** 1 in. **(E)** 2 in.

9. Mike and Dave are elves that work at Santa's Presents, where they, together, need to pack 126 presents for Christmas. If they both worked by themselves for an hour, Mike could pack 30 presents, and Dave could pack 50. Dave, to Mike's disappointment, arrives 12 minutes late, while Mike arrives right on time. If both start packing at the exact time they arrive, how many presents does Dave pack?

(A) 50 **(B)** 65 **(C)** 75 **(D)** 85 **(E)** 100

10. The average of the reciprocals of two positive numbers is less than $\frac{1}{2}$. If one of the numbers is twice the other, then the other number must be less than the first by at least more than $\frac{a}{b}$, a fraction that is both as large as possible, and in its simplest form. What is $a + b$?

(A) 5 **(B)** 6 **(C)** 9 **(D)** 10 **(E)** 12

11. A naturalist wants to test his theory that all leaves that come from the same species are similar; that is, any one leaf from a given species can be dilated (either scaled up or scaled down) to obtain any other leaf in that same species. The naturalist finds a leaf on the ground of area $100\ \text{mm}^2$ that has exactly two dark spots with a distance of 8 mm between them. He has another leaf of the same species at his lab with a distance of 10 mm between its dark spots. If his finding supports his theory, what is the area of the leaf at his lab?

(A) $156\,\text{mm}^2$ **(B)** $156\frac{1}{4}\,\text{mm}^2$ **(C)** $175\,\text{mm}^2$ **(D)** $181\frac{1}{4}\,\text{mm}^2$ **(E)** $187\frac{1}{2}\,\text{mm}^2$

12. Al, Bob, and Carl each have favorite numbers so that the sum of Al and Bob's favorite numbers has a units digit of 2, the sum of Bob and Carl's favorite numbers has a units digit of 4, and the sum of Al and Carl's favorite numbers has a units digit of 0. If their favorite numbers are all positive integers, what is the sum of all possible values of the units digit of the product of all three of their favorite numbers?

(A) 2 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

13. A line containing the points $(a, 0)$ and $(0, b)$ also contains the point $\left(\frac{a}{2}, \frac{a}{3}\right)$, for positive numbers a and b, b also being an integer. What is the smallest possible value of $a + b$?

(A) $\frac{1}{2}$ **(B)** 1 **(C)** $1\frac{1}{2}$ **(D)** 2 **(E)** $2\frac{1}{2}$

14. If A is a digit, then what is the smallest possible value of the sum $\underline{A12} + \underline{1A2} + \underline{12A}$ of three 3-digit numbers, such that this sum is a multiple of 7?

(A) 343 **(B)** 455 **(C)** 567 **(D)** 784 **(E)** No such A exists

15. Define the *dev* between two days to be the smallest number of days it takes to get from one of the days to the other, including both days whose dev is being taken (where "day" refers to the seven days of the week). For example, the dev between Wednesday and Sunday is 4, as the quickest way to get between them is {Sunday, Monday, Tuesday, Wednesday}, a set consisting of 4 days. Moreover, define the *average dev* of a day d to be the average of all of the devs of d, taken with respect to all of the other days. Which day has the smallest average dev?

(A) Monday **(B)** Tuesday **(C)** Wednesday **(D)** Thursday **(E)** No such day exists

16. A recursive sequence $\{a_n\}$ has $a_1 = 2$ and $a_n = 3a_{n-1} + 1$, for all $n \geq 2$. What is the units digit of $a_1 + a_2 + \cdots + a_{2018}$?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

17. Two distinct vertices of a regular hexagonal prism with all edges of length 1 are chosen at random. The length of the line segment that connects them is then measured. What is the probability that the measured length is greater than 1?

(A) $\frac{7}{11}$ **(B)** $\frac{8}{11}$ **(C)** $\frac{17}{21}$ **(D)** $\frac{6}{7}$ **(E)** $\frac{10}{11}$

18. How many distinct two-element subsets can be chosen from the set $\{1, 2, \ldots, 100\}$ of the first 100 positive integers, such that the two elements in the chosen subset sum to a multiple of 5?

(A) 570 **(B)** 780 **(C)** 990 **(D)** 1000 **(E)** 1200

19. In isosceles $\triangle ABC$, $AB = AC = 2$ and $BC = 1$. Let D and E be the points where the bisector of $\angle B$ and the median of the triangle from vertex B intersect $\overline{AC}$, respectively. What is the area of $\triangle BDE$?

(A) $\frac{\sqrt{15}}{24}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{\sqrt{15}}{4}$ **(D)** 2 **(E)** $2\sqrt{5}$

20. The following repeating sequence is created by listing symbols:

$$\{\heartsuit, \clubsuit, \spadesuit, \diamondsuit, \circ, \heartsuit, \clubsuit, \spadesuit, \diamondsuit, \circ, \heartsuit, \clubsuit, \spadesuit, \diamondsuit, \circ, \ldots\}.$$

What is the 2018^{2018}th symbol in this sequence?

(A) $\heartsuit$ **(B)** $\clubsuit$ **(C)** $\spadesuit$ **(D)** $\diamondsuit$ **(E)** $\circ$

21. A *rhombicosidodecahedron* is a polyhedron (a three-dimensional solid with many faces) constructed with 120 edges, all of the same length, forming a total of 62 faces. In particular, it has 20 triangular faces, 30 square faces, and 12 pentagonal faces. How many vertices does this polyhedron have?

(A) 30 **(B)** 60 **(C)** 120 **(D)** 240 **(E)** 480

22. The quarter-circular arc $\overset{\frown}{AC}$, centered at D, is drawn in a unit square $ABCD$. Then, the radius of the largest circle that can fit in the region enclosed between $\overline{AB}$, $\overline{BC}$, and $\overset{\frown}{AC}$ can be expressed in simplest radical form as $a - b\sqrt{c}$, for positive integers a, b, and c. What is $a + b + c$?

(A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 10

23. The *binary* counting system uses only two digits, 0 and 1, instead of the ten $(0, 1, \ldots, 9)$ that are used in the common base-10 decimal number system today. As a result, counting in binary is done as $0, 1, 10, 11, 100, 101, 110, 111, 1000$, and so on, so that each integer in the counting sequence is the smallest one higher than the previous that can be created with only 0's and 1's. John writes down a list of the first 100 positive integers in binary. How many total digits did he write down?

(A) 440 **(B)** 580 **(C)** 760 **(D)** 972 **(E)** 1160

24. Rectangle $ABCD$ is drawn with a point E located on $\overline{CD}$ with ratio $CE : DE : AE = (2 - \sqrt{3}) : 1 : 2$, along with a point F located on $\overline{BC}$ such that $BF : FC = (3 - \sqrt{3}) : (2\sqrt{3} - 3)$. What is $\text{m}\angle EAF$?

(A) 15° **(B)** 30° **(C)** 45° **(D)** 60° **(E)** 75°

25. There is exactly one unique quintuplet of real numbers (a, b, c, d, e) that satisfies the following system of equations:

$$\begin{gathered} a + b + c + d + e = -10 \\ 8a + 4b + 2c + d + 2e = 1 \\ 27a + 9b + 3c + d + 3e = 2 \\ 216a + 36b + 6c + d + 4e = 5 \\ c + d = e \end{gathered}$$

Coincidentally, the value of $343a + 49b + 7c + d + 5e$ is some integer N. What is the sum of the digits of $|N|$?

(A) 5 **(B)** 8 **(C)** 10 **(D)** 13 **(E)** 16

3 Credits

This test was written and formatted by mathchampion1, and the two test solvers were PiMath12345 and kcbhatraju.