popcorn1's AMC 8 2018 (Rules)

Rules

READ ALL OF THESE RULES BEFORE CONTINUING.

1. DO NOT OPEN THIS BOOKLET UNTIL YOU HAVE SET YOUR TIMER TO 40 MINUTES.
2. This is a twenty-five question multiple choice test. For each question, only *one* answer choice is correct.
3. Mark your answer to each problem on the popcorn1's AMC 8 Answer Form. You may want to mark your answers on paper and then, after the test is over, submit your answers using the form.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only scratch paper, graph paper, rulers, protractors, and erasers are allowed as aids. Calculators are NOT allowed. No problems on the test require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 40 minutes to complete the test once you start your timer.
8. When you finish the exam, put your AoPS username in the space provided on the answer form.

BY CONTINUING, YOU AGREE TO ALL OF THE ABOVE RULES.

Contributors: popcorn1

1 Which of these numbers is the same upside down?

(A) 68 **(B)** 69 **(C)** 70 **(D)** 71 **(E)** 72

2 Three distinct positive integers have a product of 15 and a sum of n. Find n.

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 17

3 How many rearrangements of the digits of the number 2018 do not result in a four-digit number?

(A) 0 **(B)** 4 **(C)** 6 **(D)** 12 **(E)** 18

4 Which of the options is the sum of the other four?

(A) -26 **(B)** -6 **(C)** 12 **(D)** 20 **(E)** 24

5 $W, X, Y,$ and Z are four collinear points. The table below shows some of the distances between points.

Line Segment	Length
XZ	5
WY	6
XW	3
WZ	2
ZY	4

Find the longest distance between any two of these four points.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

6 A three-digit positive integer has the product and sum of its digits both equal to n. What is n?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

7 Which of these is the largest?

(A) 2018 **(B)** 2^{018} **(C)** 20^{18} **(D)** 201^{8} **(E)** $2^0 \times 1^8$

8 The center of a square is $\sqrt{2}$ units away from one of the vertices. What is its area?

(A) 1 **(B)** 4 **(C)** 9 **(D)** 16 **(E)** 25

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9 For distinct positive integers A, M, and C, it is known that $A + M + C = 8$. Find the maximum possible value of $A \times M \times C$.

(A) 10 **(B)** 12 **(C)** 15 **(D)** 16 **(E)** 18

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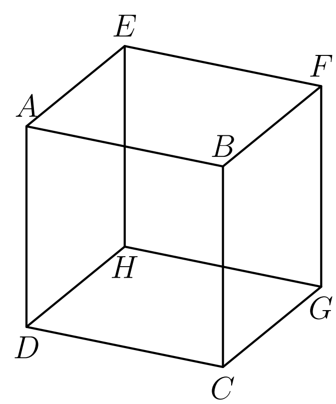
10 The ratio of the area of one square to another square is $4 : 9$. One of the squares has a perimeter of 12. Find the maximum possible perimeter of the other square.

(A) 8 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 27

11



In the unit cube shown above, how many distinct paths are there to get from A to G by only walking along the edges and walking a total of 3 units?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

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12 A square of area 100 is cut into four rectangles. What is the maximal sum of the four rectangles' perimeters?

(A) 40 **(B)** 60 **(C)** 80 **(D)** 96 **(E)** 100

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13 60 adults were surveyed on whether they preferred football or basketball. The ratio of women who said they liked football to the number of women who said they liked basketball was $4 : 5$, and the ratio of males to females surveyed was

1 : 3. The number of people who preferred football was a perfect cube. How many men liked basketball?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

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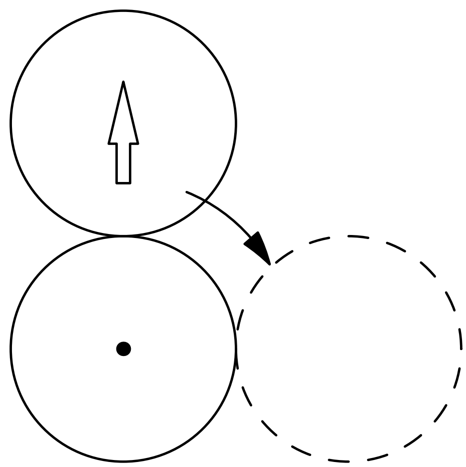
14 In the diagram below, the top circle of radius 1 is rolled around the bottom circle of radius 1 until it goes into the dotted outline. When the top circle stops in the dotted outline, which direction is the arrow on the circle pointing?



(A) up **(B)** down **(C)** left **(D)** right **(E)** It depends on the speed of rotation

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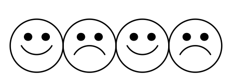
15 Four buttons are in a row as shown. When you push a button, the button and all buttons adjacent to it invert (happy becomes sad and vice versa.) Find the *fewest* number of buttons one needs to push so that all the faces are happy.



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

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16 Seven coins are in a row, all heads up. How many ways can you flip three of them so that no two coins showing tails are adjacent to each other? The order in which the coins are flipped does not matter.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

17 Leo has nine cards with the integers from 1 to 9, inclusive. He picks two of the nine cards. The probability that their product is a prime number is $\frac{n}{72}$. What is n?

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 20

18 In convex quadrilateral $WXYZ$, $WX = 6$, $XY = 8$, $YZ = 15$, and the perimeter is $29 + 5\sqrt{13}$. $\angle WXY = 90^\circ$. Find the area of the quadrilateral.

(A) $\frac{99}{2}$ **(B)** 50 **(C)** 99 **(D)** 100 **(E)** 198

19 There is a three digit number $\overline{abc}$ such that $a! + b! + c! = \overline{abc}$. Find $a + b + c$. Note: $\overline{abc}$ represents the three digit number formed by placing the digits a, b, c next to each other, rather than their product. $N!$ is the product of all integers from 1 to N, inclusive.

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

20 Let e be the number of even factors of 3210 and o be the number of odd factors of 3210. Find the value of $\frac{o}{e}$.

(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** 1

21 In equilateral triangle ABC, B is the midpoint of AD and C is the midpoint of DE. Find the length AE, given that $AB = 1$.

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $2\sqrt{3}$

22 Four real numbers a, b, c, and d satisfy $a + 1 = b - 2 = c^2 + 3 = d^2 - 4$. Which of them is the largest?

(A) a **(B)** b **(C)** c **(D)** d **(E)** Multiple answers are possible.

23 Simon and Gary are playing a game. Gary picks a three-digit number with all digits different and no digits equal to zero. Then, Simon tells Gary what number he thinks it is and Gary tells him how many digits are correct. For example, if Gary's number is 234 and Simon says 354, Gary would tell Simon 2. The game continues until Simon's guess matches Gary's number exactly. For one game, the dialogue went as follows:

Simon: 628.
Gary: 2.

Simon: 638.
Gary: 2.

What is the *fewest* number of additional guesses Simon must do to be *absolutely sure* he knows Gary's number? The final guess where Simon says Gary's number counts as a guess.

(A) 7 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

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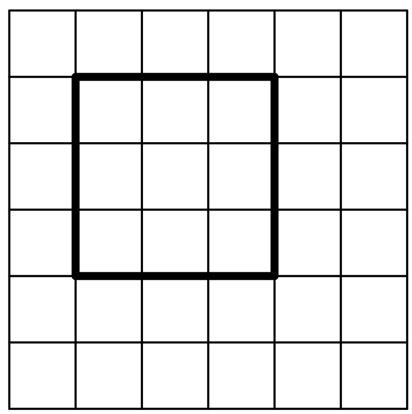
24 Thirty-six red, blue, and green beads are in the ratio $1:2:3$. They are then placed in a 6×6 square, with one bead per square and such that if two beads shared the same color, their cells did not share a side. How many different possibilities are there for the arrangement of beads in the 3×3 square shown? Note: two arrangements are considered different if they are not identical. Rotations and reflections are considered different.



(A) 16 **(B)** 24 **(C)** 32 **(D)** 48 **(E)** 54

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25 David has a paper rectangle $ABCD$ such that $AB = 2$ and $BC = 1$. He folds it such that A lands on C. Then he folds it such that B lands on D. Then he traces the resulting shape onto a piece of paper. What is the area of the shape David traced?

(A) $\frac{9}{16}$ **(B)** $\frac{5}{8}$ **(C)** $\frac{11}{16}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{13}{16}$