1. Compute the quantity $99^2 + 2 \times 99 + 1$.

$\textbf{(A)}\ 9604 \qquad \textbf{(B)}\ 9801 \qquad \textbf{(C)}\ 9999 \qquad \textbf{(D)}\ 10000 \qquad \textbf{(E)}\ 10201$

2. Consider a base 10 positive integer to be $cute$ if it has the same number of digits when expressed in base 4. How many $cute$ integers are there?

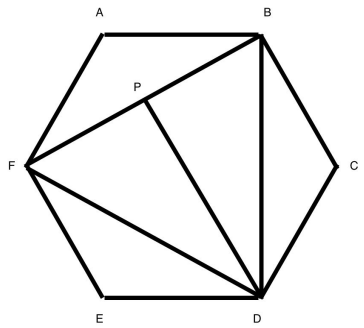
$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 11 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ \text{infinitely many}$

3. For which value of b does the system of equations

$2x + 4y = 20$ and $bx + 9y = 45$ have an infinite number of solutions?

$\textbf{(A)}\ 4 \qquad \textbf{(B)}\ 4.5 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 5.5 \qquad \textbf{(E)}\ 6$

4. In the figure below, $ABCDEF$ is a regular hexagon with side length 1, and P is the midpoint of BF. Find the length of PD.



$\textbf{(A)}\ \sqrt{3} \qquad \textbf{(B)}\ \frac{3}{2} \qquad \textbf{(C)}\ \frac{3}{4} \qquad \textbf{(D)}\ 1 \qquad \textbf{(E)}\ \frac{\sqrt{3}}{2}$

5. Find the units digit of $1 + 2 + 2^2 + 2^3 + 2^4 + ... + 2^{2019}$.

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 5 \qquad \textbf{(D)}\ 7 \qquad \textbf{(E)}\ 9$

6. How many ordered pairs of digits (a, b) satisfy the requirement that the number $16ab2$ is divisible by 9?

$\textbf{(A)}\ 9 \qquad \textbf{(B)}\ 10 \qquad \textbf{(C)}\ 11 \qquad \textbf{(D)}\ 12 \qquad \textbf{(E)}\ 13$

7. Markus has to buy some DVDs for his younger brother's birthday party. The Avengers DVDs cost 8 dollars each, while the Batman DVDs cost 12 dollars each. Markus has 60 dollars to spend. Find the number of ways he can buy the DVDs such that he spends all his money.

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 3 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 5 \qquad \textbf{(E)}\ 6$

8. How many ways are there for Anna, Ben, and Claire to choose 3 chairs out of a row of 9 chairs to sit on such that at least one pair of adjacent chairs contain two people? (For example, a valid way is AB-C - - - - -)

$\textbf{(A)}\ 168 \qquad \textbf{(B)}\ 210 \qquad \textbf{(C)}\ 294 \qquad \textbf{(D)}\ 384 \qquad \textbf{(E)}\ 504$

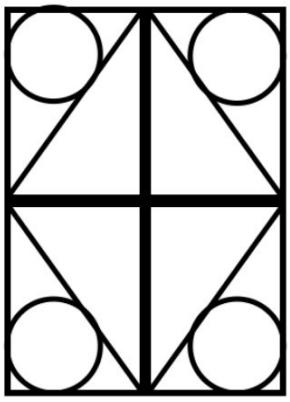
9. How many arrangements of the number 12569 result in a multiple of 11?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 6 \qquad \textbf{(C)}\ 12 \qquad \textbf{(D)}\ 24 \qquad \textbf{(E)}\ 36$

10. A ball is thrown from a cliff of height 20 meters. Every time it hits the ground, it bounces to $\frac{1}{2}$ of its peak height before it hits the ground and then falls back down. Find the number of meters the ball travels.

$\textbf{(A)}\ 40 \qquad \textbf{(B)}\ 45 \qquad \textbf{(C)}\ 50 \qquad \textbf{(D)}\ 55 \qquad \textbf{(E)}\ 60$

11. A window in a cathedral is made out of 8 congruent right triangles with circles inscribed inside 4 of them, as shown in the diagram. Given that the total area of the window is 48 square feet and that the sides of the triangles are all positive integers, find the area of the rectangle with their four vertices as the centers of the four circles.



$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 24 \qquad \textbf{(C)}\ 30 \qquad \textbf{(D)}\ 32 \qquad \textbf{(E)}\ 35$

12. How many four digit positive integers have the property that the product of their digits is 120?

$\textbf{(A)}\ 60 \qquad \textbf{(B)}\ 66 \qquad \textbf{(C)}\ 72 \qquad \textbf{(D)}\ 84 \qquad \textbf{(E)}\ 96$

13. Consider the sequence of positive perfect cubes $1, 8, 27, 64, 125, 216, 343, \ldots$. Let Set A contain all the differences between adjacent numbers in the sequence. Let Set B contain all the differences between adjacent elements in Set A. Find the sum of the first 50 elements in Set B.

$\textbf{(A)}\ 7800 \qquad \textbf{(B)}\ 7950 \qquad \textbf{(C)}\ 8100 \qquad \textbf{(D)}\ 8262 \qquad \textbf{(E)}\ 15300$

14. A cone has a volume of 216π and has radius r and height h. How many ordered pairs of integers (r, h) are possible?

$\textbf{(A)}\ 2 \qquad \textbf{(B)}\ 4 \qquad \textbf{(C)}\ 6 \qquad \textbf{(D)}\ 8 \qquad \textbf{(E)}\ 10$

15. A child builds towers using identically shaped cubes of different color. How many different towers of height 9 cubes can he build with 3 red cubes, 3 green cubes, and 3 blue cubes such that no two red cubes are adjacent to each other and that all the green cubes are adjacent to each other?

$\textbf{(A)}\ 38 \qquad \textbf{(B)}\ 39 \qquad \textbf{(C)}\ 40 \qquad \textbf{(D)}\ 41 \qquad \textbf{(E)}\ 42$

16. How many positive integers less or equal to 50 satisfy the property that their square has exactly 9 positive factors?

$\textbf{(A)}\ 11 \qquad \textbf{(B)}\ 12 \qquad \textbf{(C)}\ 13 \qquad \textbf{(D)}\ 14 \qquad \textbf{(E)}\ 15$

17. How many ordered quadruples of positive integers (a, b, c, d) where $1 \leq a, b, c, d \leq 4$ satisfy the property that $a + b + c + d$ is a multiple of 4?

$\textbf{(A)}\ 62 \qquad \textbf{(B)}\ 64 \qquad \textbf{(C)}\ 70 \qquad \textbf{(D)}\ 72 \qquad \textbf{(E)}\ 142$

18. A real number a is chosen at random in the range $[-100, 100]$. Find the probability that the equation $x^2 + (4 + a)x + a^2$ has two real roots.

$\textbf{(A)}\ \frac{1}{75} \qquad \textbf{(B)}\ \frac{2}{150} \qquad \textbf{(C)}\ \frac{4}{150} \qquad \textbf{(D)}\ \frac{1}{25} \qquad \textbf{(E)}\ \frac{8}{150}$

19. Given that positive integers a, b, c, d satisfy $a + b + c + d = 17$, what is the maximum possible product $abcd$?

$\textbf{(A)}\ 240 \qquad \textbf{(B)}\ 250 \qquad \textbf{(C)}\ 288 \qquad \textbf{(D)}\ 300 \qquad \textbf{(E)}\ 320$

20. How many ordered quintuplets of integers (p, q, r, s, t) have the property that $pqrst = -2019^3$?

$\textbf{(A)}\ 1225 \qquad \textbf{(B)}\ 18375 \qquad \textbf{(C)}\ 19600 \qquad \textbf{(D)}\ 73500 \qquad \textbf{(E)}\ 78400$

21. For how many integers n where $0 \leq n \leq 2019$ is $n^2 + 1$ a multiple of 25?

$\textbf{(A)}\ 80 \qquad \textbf{(B)}\ 82 \qquad \textbf{(C)}\ 160 \qquad \textbf{(D)}\ 162 \qquad \textbf{(E)}\ 164$

22. Four distinct elements are randomly selected from the set $2019, 2019^2, 2019^3, 2019^4, 2019^5, 2019^6$. The probability that their sum is divisible by 13 can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

$\textbf{(A)}\ 6 \qquad \textbf{(B)}\ 7 \qquad \textbf{(C)}\ 16 \qquad \textbf{(D)}\ 17 \qquad \textbf{(E)}\ 19$

23. Compute the remainder when 11^{41} is divided by 1000.

$\textbf{(A)}\ 011 \qquad \textbf{(B)}\ 211 \qquad \textbf{(C)}\ 411 \qquad \textbf{(D)}\ 611 \qquad \textbf{(E)}\ 811$

24. There are four girls, Patricia, Dorothy, Mandy, and Sarah, that stand in a line with 10 boys in that order. Let p be the number of boys standing between Patricia and Dorothy, d the number of boys standing between Dorothy and Mandy, and m the number of boys standing between Mandy and Sarah. The number of ways there are to arrange the line where the product pdm is even can be expressed as $10! \times a$, where a is a positive integer. For example, $PBDBBMBBBBBBBS$ is an acceptable arrangement. Find a. (Note that if two girls are standing adjacent to each other than the number of boys standing between them is 0.)

$\textbf{(A)}\ 931 \qquad \textbf{(B)}\ 939 \qquad \textbf{(C)}\ 953 \qquad \textbf{(D)}\ 971 \qquad \textbf{(E)}\ 981$

25. There are 10 cards laid out face down in a row: $8, 9, 10, J, Q, K, A, 2$, and the two Jokers. Every 15 seconds Matt randomly chooses three distinct cards to flip over. Find the expected value of the time he takes to flip over one of the Jokers for the first time.

$\text{(A)}\ \frac{225}{8} \qquad \text{(B)}\ \frac{225}{7} \qquad \text{(C)}\ 45 \qquad \text{(D)}\ \frac{225}{4} \qquad \text{(E)}\ \frac{450}{7}$