You will have 40 minutes to answer 25 questions. To submit your answers, either PM me or use the Google Forms that will be provided in the thread. Note that the Google Forms will automatically display your score once you are finished with the test.

1. If $a\spadesuit b = a^2 - ab$, find the value of $69\spadesuit 68$.

(A) 67 **(B)** 68 **(C)** 69 **(D)** 136 **(E)** 138

2. How many ways are there to seat three people, P_1, P_2, and P_3 around a circular table with three seats such that rotations, but not reflections, of an arrangement count as the same arrangement?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 6

3. There is a real number n such that the numerical values of sum of the edge lengths and the surface area of a cube with side length n are equal. Find n.

(A) 1 **(B)** $\sqrt{2}$ **(C)** 2 **(D)** 3 **(E)** 4

4. Abe, Bill, Connor, Derek, and Ethan stand in a line such that Ethan stands either in the frontmost spot or the backmost spot. Find the number of ways this can happen.

(A) 12 **(B)** 24 **(C)** 36 **(D)** 48 **(E)** 72

5. Find the least integer n such that $n!$ is divisible by 10^6.

(A) 20 **(B)** 25 **(C)** 30 **(D)** 35 **(E)** 40

6. Find the units digit of $1 + 2 + 3 + 4 + ... + 2019$.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

7. Two distinct numbers (x_1, x_2) are randomly selected from the set $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14$. Find the probability that $|x_1 - x_2| > 4$.

(A) $\frac{36}{91}$ **(B)** $\frac{45}{91}$ **(C)** $\frac{55}{91}$ **(D)** $\frac{45}{98}$ **(E)** $\frac{55}{98}$

8. Adrienne, Bill, Carson, and Danielle are randomly assigned seats around a round table that has four seats. Find the probability that Adrienne does not sit adjacent to Bill.

(A) $\frac{1}{24}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{2}{3}$

9. An integer n is selected at random from the range $[-1000, 999]$. Find the probability that 3^n leaves a remainder of 1 when divided by 10.

(A) $\frac{249}{1000}$ **(B)** $\frac{499}{2000}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{501}{2000}$ **(E)** $\frac{251}{1000}$

10. Let S be the set of all triangles that can be formed from three distinct vertices of a regular decagon (or 10 sided polygon). How many of those triangles are scalene?

(A) 60 **(B)** 70 **(C)** 80 **(D)** 90 **(E)** 100

11. Al and Bob are two swimmers that both start swimming at the same side of the bulkhead (or the same end of the lane). They start swimming at the exact same time and swim in the same direction, and while they swim, if they run into a bulkhead (or end of an lane), they immediately flip-turn back to swim in the opposite direction. It is known that Al's constant speed is 2 yards per second and that when he finishes his set of 12 50's Bob's position is exactly at the opposite bulkhead of the bulkhead that Al finished on. (Assume that the length from bulkhead to bulkhead is 25 yards.) Find the sum of all possible values of Bob's speed in yards per second.

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

12. Let R be the set of all points on the graph of $y = \pm|\frac{1}{3}x|$ such that $-10 \leq x \leq 10$. Let C be the circle completely containing all of those points. Find the radius of C.

(A) 10 **(B)** $\frac{5\sqrt{37}}{3}$ **(C)** $\frac{10\sqrt{10}}{3}$ **(D)** $3\sqrt{10}$ **(E)** $\frac{20\sqrt{3}}{3}$

13. How many five digit positive integers have distinct digits $(d_1, d_2, d_3, d_4, d_5)$ such that $d_1 > d_2 > d_3 > d_4 > d_5$?

(A) 126 **(B)** 210 **(C)** 252 **(D)** 1287 **(E)** 2002

14. A particle starts at the coordinate point $(0,0)$ and travels to the coordinate point $(3,3)$ such that on each step, it may either travel from (x,y) to $(x+1,y)$, $(x+2,y)$, $(x+3,y)$, $(x,y+1)$, $(x,y+2)$, or $(x,y+3)$. Find the number of distinct sequences of steps that the particle can take.

(A) 45 **(B)** 62 **(C)** 88 **(D)** 94 **(E)** 106

15. Let the function $f(n)$ be defined recursively by $f(n+2) = 2 \times f(n) + f(n+1)$ for all integers $n \geq 2$. Given that $f(0) = f(1) = 1$, compute the remainder when $f(111)$ is divided by 7.

(A) 0 **(B)** 1 **(C)** 3 **(D)** 4 **(E)** 5

16. Find the remainder when $7^0 + 7^1 + 7^2 + 7^3 + ... + 7^{2019}$ is divided by 19.

(A) 0 **(B)** 1 **(C)** 7 **(D)** 8 **(E)** 18

17. There is a unique positive integer x such that $x^2 + 2x + 69 = n^2$, where n is a positive integer. Find the sum of the digits of x.

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

18. A bar sells three types of beer: Beer A, Beer B, and Beer C, in which each bottle costs 2 dollars, 2 dollars, and 3 dollars, respectively. Ricardo buys $a \geq 1$ bottles of Beer A, $b \geq 1$ bottles of Beer B, and $c \geq 1$ bottles of Beer C and spends 100 dollars total. Find the number of possible ordered triples (a,b,c).

(A) 329 **(B)** 376 **(C)** 408 **(D)** 425 **(E)** 459

19. How many ways are there to tile a 3×12 floorboard with 4 3×2 tiles and 12 1×1 tiles such that all tiles are used and that no two tiles are overlapping? (Rotations and reflections matter in this problem.)

(A) 70 **(B)** 321 **(C)** 838 **(D)** 871 **(E)** 886

20. Let $a_i = 1010010001...\underbrace{000...000}_{\text{i 0s}}1$. For how many values of i in the range $1 \leq i \leq 1000$ is a_i divisible by 99?

(A) 26 **(B)** 27 **(C)** 28 **(D)** 29 **(E)** 30

21. Which one of these integers is not a perfect square?

(A) $3,164,517,230,281$
(B) $8,322,076,809,601$
(C) $9,999,622,679,524$
(D) $26,944,165,863,029$
(E) $95,862,799,812,025$

22. Four distinct positive integers (a_1, a_2, a_3, a_4) are randomly chosen from the set $1, 2, 3, 4, ..., 12$. Find the probability that the product $a_1a_2a_3a_4$ is divisible by 20.

(A) $\frac{41}{99}$ **(B)** $\frac{43}{99}$ **(C)** $\frac{5}{11}$ **(D)** $\frac{47}{99}$ **(E)** $\frac{49}{99}$

23. Consider Triangle ABC, with $AB = 48$, $BC = 64$, and $CA = 80$. Cevian BX cuts AC into two segments of equal length. The incircles of Triangles BCX and BAX touch BX at Points P and Q, respectively. Find PQ.

(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

24. Compute the remainder when $2019^{2018^{2017^{2016}}}$ is divided by 1000.

(A) 041 **(B)** 241 **(C)** 441 **(D)** 641 **(E)** 841

25. What is the largest three digit positive integer such that $3^n - 1$ is divisible by 11^3?

(A) 975 **(B)** 980 **(C)** 985 **(D)** 990 **(E)** 995