# Rules
Read all of these rules before continuing.

1. The following test consists of 25 problems on 3 pages to be completed in 40 minutes. Each question is followed by answers labeled A, B, C, D, E. Only one of these answers is correct.

2. The answers to the problems are to be marked on the popcorn1's AMC 8 A 2019 Answer Form. Only properly marked answers will be graded.

3. There is no penalty for guessing. Your score is the number of correct answers.

4. Figures are not necessarily drawn to scale, unless otherwise mentioned.

5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, computing devices, or resources such as Wolfram-Alpha are allowed. No problems on the exam require the use of a calculator.

6. When you feel like it, begin working on the problems. You will have **40 minutes** to complete the exam[1].

By continuing, you have read and agree to all the rules on this page.

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[1]popcorn1 and others who work on the exam reserve the right to disqualify scores from an individual if they determine that the required examination procedures were not followed or if any sort of cheating has occurred.

1. What is the value of $2^2 + 0^2 + 1^2 + 8^2$?
   (A) 69 (B) 70 (C) 71 (D) 72 (E) 73

2. How many real solutions does the equation $x^3 = 1^x$ have?
   (A) 0 (B) 1 (C) 2 (D) 3 (E) Infinitely many

3. What is the ratio of the largest 8-digit integer to the largest 4-digit integer?
   (A) 1001 (B) 1111 (C) 10,001 (D) 11,111 (E) 100,001

4. A video on YouTunnel gets 10 views every second. If k means thousand (i.e. $10k$ means 10,000) and similarly m means million, then which of these is closest to the number of views it gets in a year?
   (A) $300k$ (B) $3m$ (C) $30m$ (D) $300m$ (E) $3000m$

5. The bar graph below shows how many glasses of water Marie drank each day over the month of October.
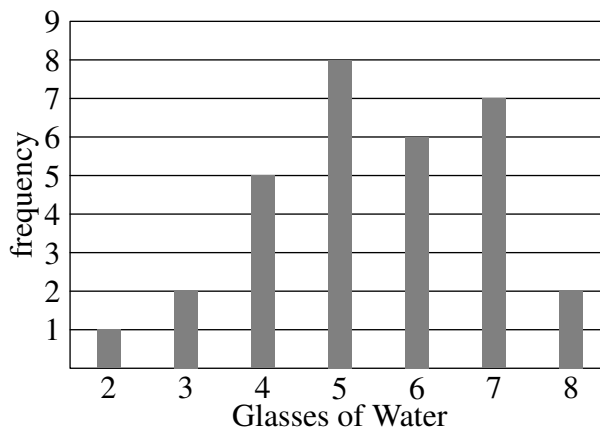
   If Marie selected a random day in October, what is the probability she drank at least 6 glasses of water on that day, to the nearest whole percent?
   (A) 29% (B) 32% (C) 48% (D) 52% (E) 71%

6. A square floor is tiled completely with square tiles, measuring 2 feet on a side. There are 36 tiles touching the walls of the room. No tiles are cut. What is the area of the floor, in ft^2?
   (A) 64 (B) 100 (C) 128 (D) 256 (E) 400

7. The triangle and square shown share a side. Their areas are both equal to 4. What is the height of tower (the dashed line in the picture)?
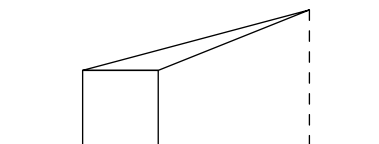

   (A) 4 (B) $4\sqrt{2}$ (C) 6 (D) $6\sqrt{2}$ (E) 8
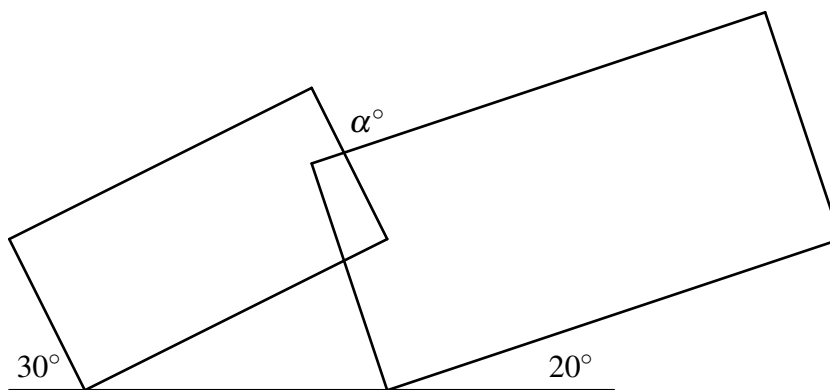
8. Two rectangles are falling over, as shown. What is the measure of the angle α, in degrees?
   (**A**) 70   (**B**) 120   (**C**) 130   (**D**) 150   (**E**) 170

9. Dave had no money. Then Andrew gave Dave $\frac{3}{5}$ of his money, Bob $\frac{2}{7}$, and Carl $\frac{1}{4}$. Each person gave Dave \$17. Who has the most money now?
   (**A**) Andrew   (**B**) Bob   (**C**) Carl   (**D**) Dave   (**E**) There is a tie.

10. A class is playing the game *Headbandzz*. In the game, everyone wears a red, green, or blue headband. Every student can see every headband except for their own. Then, every student truthfully says that they see at least 20 red headbands, 1 blue headband, and 9 green headbands. What is the fewest number playing the game?
    (**A**) 27   (**B**) 30   (**C**) 33   (**D**) 36   (**E**) 39

11. A pyramid *could* have how many edges?
    (**A**) 2019   (**B**) 2019^2 (**C**) 201^9 (**D**) $201-9$ (**E**) 2019^{2019}

12. How many integers x satisfy $-20 < |x| < 19$?
    (**A**) 36   (**B**) 37   (**C**) 38   (**D**) 39   (**E**) 40

13. 1 red marble and g green marbles are placed in a jar. 7 of them are picked at random. The probability that the red marble is picked is $\frac{1}{7}$. What is the sum of the digits of g?
    (**A**) 11   (**B**) 12   (**C**) 13   (**D**) 14   (**E**) 15

14. Some cells in the 5×5 grid shown were shaded so that no 2×2 square has all of its cells shaded. What is the fewest number of cells that can be left unshaded?
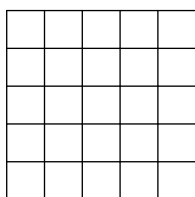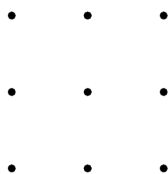

    (**A**) 3   (**B**) 4   (**C**) 5   (**D**) 6   (**E**) 7

15. Evaluate $2^{2019} + 1 + 2^{2019} + 2 + 2^{2019} + 3 + \cdots + 2^{2019} + 2^{2017} - (2^{2018} + 1) - (2^{2018} + 2) - (2^{2018} + 3) - \cdots - (2^{2018} + 2^{2017})$.
    (**A**) 2^{4034} (**B**) 2^{4035} (**C**) 2^{4036} (**D**) 2^{4037} (**E**) 2^{4038}

16. Integers 1 to 14 each used once to construct 7 fractions (for example, $\frac{10}{5}, \frac{13}{4}, \frac{7}{11}, \frac{14}{2}, \frac{9}{8}, \frac{6}{3}, \frac{1}{12}$).
    What is the maximum number of these fractions that are also integers?
    (**A**) 3   (**B**) 4   (**C**) 5   (**D**) 6   (**E**) 7

17. Find the least odd prime factor of $2019^8 - 1$.
    (**A**) 3   (**B**) 5   (**C**) 7   (**D**) 11   (**E**) 97

18. A square with side length 1 was rotated $45°$ about one of its vertices. The area of the overlap of the two squares is
    (**A**) 0.   (**B**) $1 - \frac{\sqrt{2}}{2}$.   (**C**) $\sqrt{2} - 1$.   (**D**) $\frac{1}{2}$.   (**E**) $2 - \sqrt{2}$.
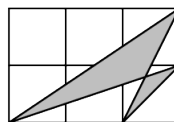
19. What fraction of the 2×3 grid of squares is shaded?
    (**A**) $\frac{5}{36}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{3}{8}$ (**D**) $\frac{7}{30}$ (**E**) $\frac{1}{4}$

20. Aaron, an ant, is at a vertex of a cube with volume 1. He can only walk along the edges. Find the shortest distance he can walk so he walks across all edges and ends at the start.
    (**A**) 12   (**B**) 13   (**C**) 14   (**D**) 15   (**E**) 16

21. A *lattice point* is a point with integer coordinates. A *nice square* is a square with all vertices on *lattice points* centered at the origin. How many *nice squares* of area 50 exist?
    (**A**) 2   (**B**) 3   (**C**) 4   (**D**) 6   (**E**) 12

22. Let $n!$ represent the product of the integers from 1 to n, inclusive. Determine the sum of the digits of the smallest integer $k > 1$ so that $\frac{23! + k}{k}$ is not an integer.
    (**A**) 5   (**B**) 6   (**C**) 9   (**D**) 10   (**E**) 11

23. Regular hexagon *HATERS* has area 1. Logan draws triangle *EAT* and Jake draws triangle *THS*. What is the area of the region common to both triangles?
    (**A**) $\frac{1}{36}$ (**B**) $\frac{1}{30}$ (**C**) $\frac{1}{24}$ (**D**) $\frac{1}{20}$ (**E**) $\frac{1}{18}$

24. For positive integers a and b, the sum of the factors of a^b is c and the product of the factors of a^b is d. It turns out that $c = \sqrt[6]{d} - 1$. What is the smallest value of $a + b$?
    (**A**) 12   (**B**) 13   (**C**) 14   (**D**) 15   (**E**) 16

25. How many (distinct) circles pass through ≥ 3 points of this grid of equally-spaced points?

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    (**A**) 46   (**B**) 52   (**C**) 54   (**D**) 58   (**E**) 84