Rules

Read all of these rules before continuing.

1. The following test consists of 25 problems on 3 pages to be completed in 40 minutes. Each question is followed by answers labeled A, B, C, D, E. Only one of these answers is correct.
2. The answers to the problems are to be marked on the popcorn1's AMC 8 B 2019 Answer Form. Only properly marked answers will be graded.
3. There is no penalty for guessing. Your score is the number of correct answers.
4. Figures are not necessarily drawn to scale, unless otherwise mentioned.
5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, computing devices, or resources such as Wolfram-Alpha are allowed. No problems on the exam require the use of a calculator.
6. When you feel like it, begin working on the problems. You will have **40 minutes** to complete the exam[1].

By continuing, you have read and agree to all the rules on this page.

[1]popcorn1 and others who work on the exam reserve the right to disqualify scores from an individual if they determine that the required examination procedures were not followed or if any sort of cheating has occurred.

1. Which of these numbers is ***not*** a factor of 2020?
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

2. It turns out that $1 \times 2 \times 3 \times 4 \times 5 = \overline{AMC}$, for some digits A, M, and C. What is $A \times M \times C$?
(**A**) 0 (**B**) 3 (**C**) 6 (**D**) 9 (**E**) 12

3. A square has an area of 144. A rectangle has the same perimeter as the square and an area of 128. What is the length of the longer side of the rectangle?
(**A**) 6 (**B**) 8 (**C**) 12 (**D**) 14 (**E**) 16

4. A 24-hour clock keeps track of time correctly, but every time it changes, the digits are scrambled. The clock is showing 03 : 99 now. How many minutes will it take until it shows 11 : 11?
(**A**) 71 (**B**) 82 (**C**) 92 (**D**) 432 (**E**) 712

5. Each of the numbers $12, 36, 54, 78$ is placed in one of the boxes in the expression below.

$$\square \div \square + \square \div \square$$

The largest possible value of the expression is k. What is the smallest integer greater than or equal to k?
(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

6. Cookie Monster eats either two or five cookies a day; in a week, he ate a prime number of cookies. Which list gives all possible values of the number of days he could've eaten 5 cookies?
(**A**) 2, 4, 6 (**B**) 1, 2, 3, 5 (**C**) 1, 3, 5 (**D**) 0, 1, 3, 5 (**E**) 1, 3, 5, 7

7. A square $ABCD$ has side length 1. Point E is picked in or on the square. What is the least possible value of $AE + BE + CE + DE$?
(**A**) 2 (**B**) $2\sqrt{2}$ (**C**) $2 + \sqrt{2}$ (**D**) $4\sqrt{2}$ (**E**) 8

8. 20% of $\sqrt{20^{20} \times 25^{25}}$ is $20^a \times 25^b$. What is $a + b$?
(**A**) 22 (**B**) 23 (**C**) 34 (**D**) 35 (**E**) 36

9. James wrote a two-digit positive integer is written on the board. Exactly one of the following statements is true about James' number:

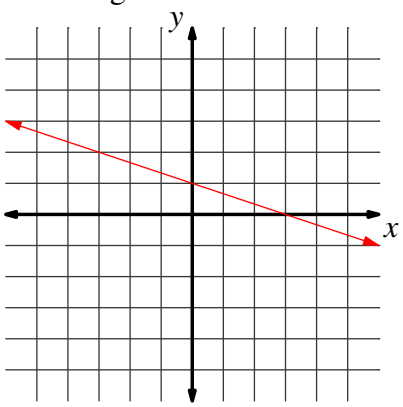
   - The units digit is 2.
   - The units digit is 3.
   - The tens digit is 4.
   - The tens digit is 5.

   How many possible numbers could James have written?
(**A**) 24 (**B**) 26 (**C**) 28 (**D**) 30 (**E**) 32

10. Consider the set $\{4, 6, 6, x\}$. The mode, median, and mean of the set are consecutive integers (not necessarily in that order). Find the range of the set.
(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

11. A $3 \times 3 \times 3$ cube was cut into $1 \times 1 \times 1$ cubes. The cube at the center of each face was removed. After these 6 cubes were removed, what is the surface area of the new solid? (Assume the cubes are glued together, and that they can be suspended in midair.)
(A) 54 **(B)** 66 **(C)** 72 **(D)** 78 **(E)** 84

12. The figure shown is to scale, but not to 1 : 1 scale. The red line's graph can be expressed in the form $x + by = c$ for some values of b and c. It turns out that $\frac{c}{b} = 2020$. What is the sum of the digits of c?



(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

13. Let k be a positive integer. The sum of the digits of k is twice the product of the digits of the k. What is the sum of the two smallest values of k?
(A) 122 **(B)** 123 **(C)** 132 **(D)** 133 **(E)** 222

14. 64% of the school has gone on a field trip. 65% of the remaining students are 6th graders. Determine the sum of the digits of the fewest number of students at the school.
(A) 1 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

15. How many ways can we shade two cells of a 4×3 grid such that they lie in the same row or the same column? (Rotations and reflections are considered distinct.)
(A) 18 **(B)** 30 **(C)** 36 **(D)** 54 **(E)** 66

16. The operation log is defined such that $\log_{(a)}(a^b) = b$. What is the value of

$$\log_{(2^1)}(2^2) \cdot \log_{(2^2)}(2^4) \cdot \log_{(2^3)}(2^6) \cdot \log_{(2^4)}(2^8) \cdot \log_{(2^5)}(2^{10})?$$

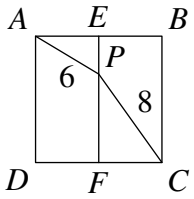
(A) 2^5 **(B)** 2^{10} **(C)** 2^{15} **(D)** 2^{30} **(E)** 2^{120}

17. We know that $3 \times 7 \times 11 \times 13 \times 37 = 111,111$. How many positive integers less than $111,111$ are multiples of four of the numbers in the set $\{3, 7, 11, 13, 37\}$?
(A) 4 **(B)** 5 **(C)** 66 **(D)** 69 **(E)** 70

18. Raj is rolling a standard six-sided die. He starts with the number 0 on the board. He rolls the die, and goes through the following procedure: if the number is prime, he will add 1 to the number on the board; if the number is even, he will add 2 to the number on the board. After 12 rolls, what is the probability he ends up with an odd number on the board?
(A) $\frac{1}{3}$ **(B)** $\frac{2047}{4096}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2049}{4096}$ **(E)** $\frac{2}{3}$

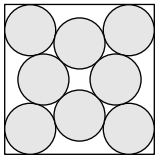
19. Anh wants to sit next to Richard. David wants to sit next to Ryan. Taylor wants to sit next to Richard. In how many ways can these five sit in a row of six chairs under these conditions?
(**A**) 12 (**B**) 16 (**C**) 24 (**D**) 32 (**E**) 36

20. In rectangle $ABCD$ with $AD = 10$, let E be the midpoint of AB and F be the midpoint of CD. Point P lies on EF such that $AP = 6$ and $CP = 8$. The area of triangle BPD can be written as $\frac{a}{b}$ for positive integers a and b. Find $a+b$.



(**A**) 69 (**B**) 75 (**C**) 108 (**D**) 122 (**E**) 193

21. Radu has 3 tiles in the shape of regular polygons. One has a sides, one has b sides, and one has $a \times b$ sides. He can place these three tiles without overlapping such that any two of them share exactly one side. If $a > b > 3$, what is $a+b$?
(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

22.  8 circles of radius 1 are wedged snugly into a square of side $\sqrt{x}+\sqrt{y}+\sqrt{z}$ for integers x, y, z, as shown in the picture. Find $x+y+z$.
(**A**) 8 (**B**) 9 (**C**) 10 (**D**) 11 (**E**) 12

23. Kelsey is sitting through an hour-long class. The probability she eats a potato chip during this class is 64%. If she has an equal chance of eating a chip anytime throughout class, what is the probability she eats it in the first 30 minutes?
(**A**) 30% (**B**) 32% (**C**) 34% (**D**) 36% (**E**) 40%

24. Bo colors each of the numbers $1, 2, ..., 2019$ in such a way that all of a number's factors have different colors than it. What is the *fewest* number of colors she needs?
(**A**) 8 (**B**) 9 (**C**) 10 (**D**) 11 (**E**) 12

25. Let j and k be positive integers. If

$$j^2 = k + 10^{999} \times \overbrace{99\ldots99}^{\text{999 nines}} \times 1\overbrace{00\ldots00}^{\text{998 zeroes}}9 \times 1\overbrace{00\ldots00}^{\text{997 zeroes}}10,$$

then what is the sum of the digits of the smallest possible value of k?
(**A**) 1 (**B**) 2 (**C**) 5 (**D**) 6 (**E**) 7