## 1 Instructions

- 1. DO NOT GO PAST THIS PAGE UNTIL YOU ARE READY TO BEGIN.
- 2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
- 3. Copy your answers onto a private message after the competition is finished. Only answers on the private message will be scored.
- 4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
- 5. No aids are permitted other than blank scratch paper, rulers, compasses, and erasers. No calculators or electronic devices other than a timer are allowed. No questions will require the use of a calculator.
- 6. Figures are not necessarily drawn to scale.
- 7. You will have 40 minutes to complete the competition starting the moment you move to the next page.
- 8. When you finish the exam, send us a private message containing your answers.

## 2 AMC 8 Problems

1. To walk up a single floor in her eighteen floor apartment building, Sarah needs to take nine steps up a flight of stairs. If Sarah starts on Floor 3 and walks up 100 steps, she would end up on the flight of stairs connecting which two floors?

(A) 11 and 12 (B) 12 and 13 (C) 13 and 14 (D) 14 and 15 (E) 15 and 16

2. Abby, Barb, and Carlos each have 35, 42, and 31 trading cards respectively. If they share their trading cards equally between each other, how many more trading cards would Carlos have than before?

(A) 4 (B) 5 (C) 6 (D) 9 (E) 11

3. In triangle ABC the measure of angle A is the average of the measures of angles B and C. What is the measure of angle A?

(A)  $45^{\circ}$  (B)  $60^{\circ}$  (C)  $75^{\circ}$  (D)  $90^{\circ}$  (E)  $120^{\circ}$ 

4. A spruce tree grows by 25 feet, increasing its height by 25%. If the tree grows for a second time by 25 feet, by what percent would its height increase?

- 5. Find the sum of the digits of  $\frac{5 \times 10^{2020}}{2}$ .
  - (A) 1 (B) 2 (C) 5 (D) 7 (E) 8
- 6. Square B with side length three is attached to a side of square A with side length four, as shown in the figure below. Find the area of the shaded region.



(A) 10 (B)  $10\frac{1}{2}$  (C) 11 (D) 14 (E)  $14\frac{1}{2}$ 

7. When expressed as a decimal rounded to the nearest ten-thousandth, what is the value of  $\frac{125+3}{125\times3}$ ?

 $(A) 0.3412 \qquad (B) 0.3413 \qquad (C) 0.3414 \qquad (D) 0.3415 \qquad (E) 0.3416$ 

8. What is the value of

$$(1+2+3) - (2+3+4) + (3+4+5) - \dots - (98+99+100)?$$

$$(A) - 150$$
  $(B) - 147$   $(C) - 144$   $(D) 147$   $(E) 150$ 

- 9. Kayla writes down the first N positive integers. What is the sum of all possible values of N if Kayla writes five multiples of 13 and six multiples of 12?
  - (A) 447 (B) 453 (C) 518 (D) 525 (E) 548

10. In Murphy's seventh grade homeroom,  $\frac{7}{12}$  of the students like tennis,  $\frac{2}{3}$  of the students like badminton, and  $\frac{1}{12}$  of the students like neither. What is the minimum possible number of students who like both tennis and badminton?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 6

- 11. For how many values of N does there exist a regular N sided polygon whose vertices all lie on the vertices of a regular 24 sided polygon?
  - (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
- 12. Quadrilateral WXYZ has its vertices on the sides of rectangle ABCD with AB = 7 and BC = 5, as shown below. What is the area of quadrilateral WXYZ?



13. To drive to the supermarket, Mable drives for m miles, then drives 12 miles per hour faster for the remaining  $\frac{4}{3}m$  miles. The amount of time Mable spent driving at each of the two speeds was equal. What was Mable's average speed during her drive to the supermarket, in miles per hour?

(A) 
$$\frac{81}{2}$$
 (B)  $\frac{288}{7}$  (C) 42 (D)  $\frac{300}{7}$  (E) 50

14. Six circles of radius one are cut out of the rectangle below. What is the area of the shaded region?



- (A)  $20 6\pi$  (B)  $24 6\pi$  (C)  $28 6\pi$  (D)  $30 6\pi$  (E)  $32 6\pi$
- 15. One metronome beeps at a steady rate of 72 beeps per minute, while another metronome beeps at a steady rate of 96 beeps per minute. If both metronomes beep at the same time once, how long will it take, in seconds, until they first beep at the same time again?

(A) 
$$2\frac{1}{2}$$
 (B) 5 (C) 10 (D) 18 (E) 24

16. A square with side length two is placed on a table, forming a 30 degree angle with the table's surface. How much higher is the top vertex of the square than the table?



(A) 
$$\frac{5}{2}$$
 (B)  $\sqrt{3} + 1$  (C)  $\frac{4\sqrt{3}}{3}$  (D) 3 (E)  $\frac{3\sqrt{3}}{2} + 1$ 

- 17. Kurtis' school schedule is made up of four classes, followed by lunch, followed by three more classes. In how many ways can Kurtis arrange his schedule if two of his classes (Reading and Writing) must occur one immediately after the other?
  - (A) 600 (B) 840 (C) 1200 (D) 1440 (E) 1680
- 18. When the number 25 is added to a list of numbers with total sum S, the average of all the numbers increases by one. What is the sum of the digits of the greatest possible value of S?

19. A magician randomly picks a three digit positive integer to put into her hat and pulls out the same number with its digits in reverse order. (For example 496 would become 694 and 720 would become 27.) What is the probability the magician pulls out a multiple of 22?

(A) 
$$\frac{1}{15}$$
 (B)  $\frac{1}{18}$  (C)  $\frac{1}{20}$  (D)  $\frac{1}{25}$  (E)  $\frac{1}{30}$ 

- 20. Tyrone has three books to read in six days. He reads one-half of a single book every day. In how many ways can he finish all the books if he may not read the same book two days in a row?
  - (A) 12 (B) 18 (C) 24 (D) 30 (E) 36
- 21. There exists a circle that is tangent to AB and BC at A and C, respectively. If AB = BC = 13 and AC = 10, what is the radius of the circle?



22. For each of the distinct sets of numbers containing only positive integers between 1 and 9 inclusive, Jordan writes the sum of the numbers in that set. What is the sum of the numbers Jordan writes?

(A) 11520 (B) 11565 (C) 11610 (D) 11655 (E) 11700

23. In rectangle ABCD, the perpendicular from B to diagonal AC bisects segment CD. Which of the following is closest to  $\frac{AB}{BC}$ ?

(A) 
$$\frac{5}{4}$$
 (B)  $\frac{4}{3}$  (C)  $\frac{7}{5}$  (D)  $\frac{3}{2}$  (E)  $\frac{8}{5}$ 

- 24. How many ordered triples of positive integers (a, b, c) satisfy gcd(a, b, c) = 20 and lcm(a, b, c) = 240?
  - (A) 18 (B) 24 (C) 36 (D) 54 (E) 72
- 25. Cheyanne rolls two standard six sided dice, then repeatedly rerolls all dice which show an odd number and stops as soon as all dice show an even number. What is the probability Cheyanne stops after exactly four rounds of rerolling?

(A) 
$$\frac{61}{1024}$$
 (B)  $\frac{1}{16}$  (C)  $\frac{67}{1024}$  (D)  $\frac{9}{128}$  (E)  $\frac{29}{256}$ 

## 3 Tiebreaker

- 1. DO NOT GO PAST THIS PAGE UNTIL YOU HAVE CAREFULLY READ THE INSTRUCTIONS AND ARE READY TO BEGIN.
- 2. The tiebreaker is optional. However, choosing not to submit may place you lower on the leaderboard. This is only used to break ties between people with the same score on the AMC 8.
- 3. For each problem, you will have at most **ten minutes** to come up with a real number answer. Only simplified answers may be marked as correct. If you have an answer before ten minutes are up, stop the timer and record the amount of time you used, along with your answer, on a private message. Once you stop the timer, you may not continue to work on the problem or change your answer.
- 4. SCORING: Only the first question you answer correctly will be considered. Whoever has question A as their first correct answer will place higher than those who have question B as their first correct answer if and only if A < B. If two people have question A as their first correct answer, the person who answered with less working time will be placed higher.
- 5. Start with question one and repeat step 3 until you reach the end of the document. There are three problems in total which significantly decrease in difficulty as you move forward.
- 6. No aids are permitted other than blank scratch paper, rulers, compasses, and erasers. No calculators or electronic devices other than a timer are allowed. No questions will require the use of a calculator.
- 7. Figures are not necessarily drawn to scale.
- 8. You may submit parts of the tiebreaker any time before the submission deadline. In particular, you do not have to submit the tiebreaker the same time you submit the AMC 8.
- 9. Good Luck! Remember that these questions are significantly more difficult than those on the AMC 8.

**Reminders:** If you are reading this page, you should already have a 10-minute timer started. Come up with an answer as quickly and accurately as you can. You have ten minutes.

**Tiebreaker 1:** A whiteboard has positive real numbers 1 and m written on it. Every second, if the numbers x and y are on the whiteboard, a ghost will replace those numbers with  $|x^2 - y^2|$  and 2xy. The ghost stops once one number on the whiteboard is m times the other. For how many positive real numbers m does the ghost stop after exactly 16 seconds?

**Reminders:** If you are reading this page, you should already have a 10-minute timer started. Come up with an answer as quickly and accurately as you can. You have ten minutes.

**Tiebreaker 2:** The perpendicular bisectors of triangle ABC can be described in the coordinate plane as lines y = 0, y = x, and  $y = \sqrt{3}x$ . Given that triangle ABC has circumradius 1, find its area.

**Reminders:** If you are reading this page, you should already have a 10-minute timer started. Come up with an answer as quickly and accurately as you can. You have ten minutes.

**Tiebreaker 3:** The diagram below is constructed by attaching an equilateral triangle, a square, a regular pentagon, and a regular hexagon together. Compute the measure of the obtuse angle formed by the three red vertices.



## 4 Optional Feedback

All of these questions are optional. You can respond to none of them, all of them, or only the ones you have good responses for.

- 1. What was the difficulty of the mock in relation to other AMCs?
- 2. How was the quality of the problems? Did the test feel like an actual AMC 8, or did it just feel like a 25-question problem set with around the same difficulty as an AMC 8/10?
- 3. Which questions did you not like? Which questions did you like?
- 4. How did you feel about the tiebreaker?