Rules

Read all of these rules before continuing.

1. The following test consists of 25 problems on 5 pages to be completed in 40 minutes. Each question is followed by answers labeled (**A**), (**B**), (**C**), (**D**), (**E**). Only one of these answers is correct.

2. The answers to the problems are to be marked on the popcorn1's AMC 8 2020 Answer Form. Only properly marked answers will be graded.

3. There is no penalty for guessing. Your score is the number of correct answers.

4. Figures are not necessarily drawn to scale, unless otherwise mentioned.

5. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, computing devices, or resources are allowed. No problems on the exam *require* the use of a calculator.

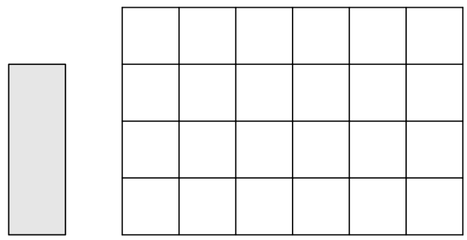
6. When you feel like it, begin working on the problems. You will have **40 minutes** to complete the exam[1].

By continuing, you have read and agree to all the rules on this page.

[1] popcorn1 and others who work on the exam reserve the right to disqualify scores from an individual if they determine that the required examination procedures were not followed or if any sort of cheating has occurred.

 # 1. What is $2020 \times 10 + 2020$?
(**A**) 20200 (**B**) 20202 (**C**) 20222 (**D**) 22220 (**E**) 22222

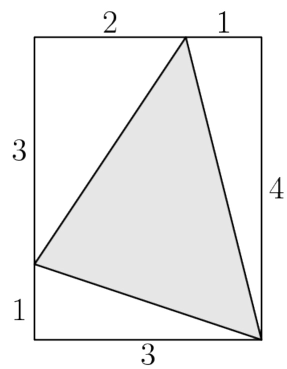
 # 2. Jake wants to cut rectangles of size 3×1 (like the grey rectangle shown) from the 4×6 grid. What is the maximum number of rectangles he can cut?



(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

 # 3. How many times does the digit 0 appear in the result of the sum $555555 + 555555$?
(**A**) 0 (**B**) 1 (**C**) 4 (**D**) 5 (**E**) 6

 # 4. A rectangle is split into four triangles, as shown. What is the area of the shaded triangle?



(**A**) 5 (**B**) 5.5 (**C**) 6 (**D**) 6.5 (**E**) 7

 # 5. The perimeter of a bicycle wheel is 108 centimeters. Approximately many rotations does the wheel make if you ride a kilometer?
(**A**) 10 (**B**) 100 (**C**) 1000 (**D**) 10,000 (**E**) 100,000

 # 6. Six cards are numbered 1, 2, ..., 6. Bartek takes some of them. Alek asks Bartek:

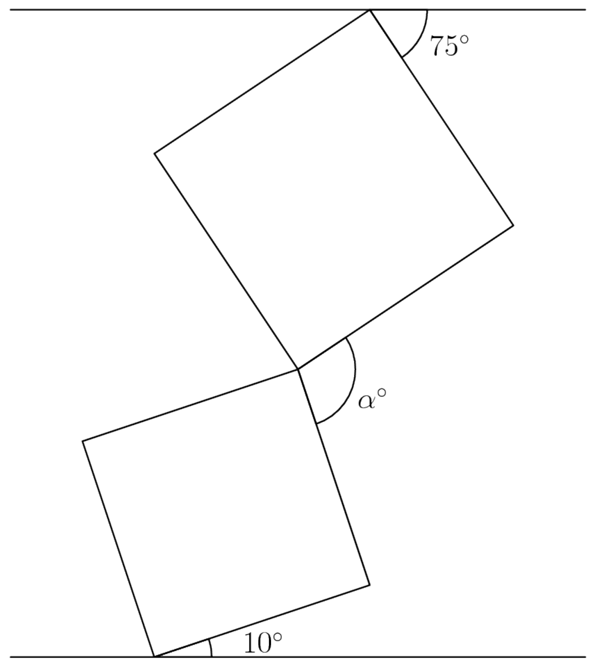
- How many of the numbers 2, 3, 4, 5, 6 do you have?
- How many of the numbers 1, 3, 4, 5, 6 do you have?
- How many of the numbers 1, 2, 4, 5, 6 do you have?
- How many of the numbers 1, 2, 3, 5, 6 do you have?
- How many of the numbers 1, 2, 3, 4, 6 do you have?

Bartek responds 4 to all of these questions. What is the sum of the numbers Bartek has?
(**A**) 15 (**B**) 16 (**C**) 17 (**D**) 18 (**E**) 19

 # 7. If $a^2 = b^3$ and $b^4 = c^5$ where a, b, and c are positive integers, then
(**A**) $a^{15} = c^8$. (**B**) $a^{16} = c^8$. (**C**) $a^{16} = c^{30}$. (**D**) $a^{64} = c^{225}$. (**E**) $a^{225} = c^{64}$.

 # 8. The average of a list of n numbers is 90. After the number $90n$ is added to the list, the average of the list is 168. Find n.
(**A**) 8 (**B**) 10 (**C**) 12 (**D**) 14 (**E**) 16

 # 9. Two squares are placed between two parallel lines, as shown. Find α.



(**A**) 85 (**B**) 95 (**C**) 100 (**D**) 125 (**E**) 150

 # 10. How many (real) solutions are there to the equation

$$\frac{(x-2)(x-3)}{(x-3)(x-4)} = -1?$$

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

 # 11. Triangles $\mathcal{S}$ and $\mathcal{T}$ overlap such that the area of the region common to both triangles is 10. The area of the region inside $\mathcal{S}$ but not $\mathcal{T}$ is 3 times the area of the region inside $\mathcal{T}$ but not $\mathcal{S}$. The area of $\mathcal{S}$ is twice $\mathcal{T}$. Find the area of $\mathcal{S}$.
(**A**) 10 (**B**) 20 (**C**) $20\sqrt{3}$ (**D**) 40 (**E**) $40\sqrt{3}$

 # 12. How many positive integers have distinct prime digits?
(**A**) 24 (**B**) 48 (**C**) 64 (**D**) 120 (**E**) 125

 # 13. Three apples were distributed to four students at random. What is the probability that no student received more than one apple?
(**A**) $\frac{1}{5}$ (**B**) $\frac{1}{4}$ (**C**) $\frac{1}{3}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{5}{6}$

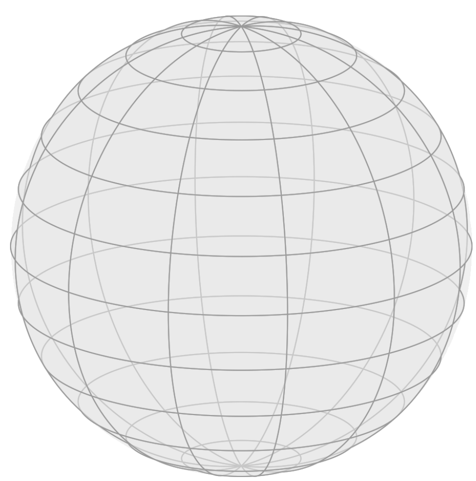
 # 14. Let

$$A = \left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{4}+\sqrt{5}\right)\cdots\left(\sqrt{2020}+\sqrt{2021}\right) \quad \text{and}$$
$$B = \left(\sqrt{2021}-\sqrt{2020}\right)\left(\sqrt{2019}-\sqrt{2018}\right)\cdots\left(\sqrt{3}-\sqrt{2}\right).$$

What is $A \cdot B$?
(**A**) 1 (**B**) $\sqrt{2021}-\sqrt{2}$ (**C**) $\sqrt{4042}$ (**D**) $\sqrt{2020\times 2021}-\sqrt{2\times 3}$ (**E**) 2020

 # 15. Let $k = 20^{20}$. Suppose that $\frac{20^k}{k^{20}} = 20^n$. Find the largest power of 20 that divides n.
(**A**) 20^2 (**B**) 20^{18} (**C**) 20^{20} (**D**) 20^{398} (**E**) 20^{400}

 # 16. A globe typically has great meridians (circles passing through the poles) and parallels (circles parallel to the equator). For example, this globe has 10 parallels and 6 great meridians.



Bela's globe has 12 parallels and 12 great meridians, breaking the surface into many parts. Bela and Jenn each pick one of these parts uniformly at random. What is the probability they pick the same part?
(**A**) $\dfrac{1}{312}$ (**B**) $\dfrac{1}{288}$ (**C**) $\dfrac{1}{156}$ (**D**) $\dfrac{1}{144}$ (**E**) $\dfrac{1}{132}$

 # 17. Niall was trying to find the value of

$$1+2+3+\cdots+2020$$

but forgot one number. His result was a multiple of 2015. Which number did he forget?
(**A**) 5 (**B**) 15 (**C**) 210 (**D**) 403 (**E**) 2015

 # 18. A concentric square and circle intersect at 8 points. In clockwise order, call these points A_1, A_2, A_3, ..., A_8. The area of $A_1A_3A_5A_7$ is $\frac{5}{8}$. Find A_2A_6.

(A) $\dfrac{\sqrt{3}}{2}$ **(B)** 1 **(C)** $\dfrac{\sqrt{5}}{2}$ **(D)** $\dfrac{\sqrt{7}}{2}$ **(E)** $\sqrt{3}$

 # 19. Bob has a rectangular piece of paper with integer side lengths. It turns out that the length of the diagonal is also an integer. The largest square Bob can draw on his paper has side length 20. What is the smallest possible length of the diagonal of Bob's paper?

(A) 20 **(B)** 25 **(C)** 29 **(D)** 52 **(E)** 101

 # 20. Seven coins lie on a table. Five of them are fair, but two coins have heads on both sides. Kai picks a coin and flips it. It lands on heads. If he flips the same coin, what is the probability it lands on heads again?

(A) $\dfrac{5}{14}$ **(B)** $\dfrac{1}{2}$ **(C)** $\dfrac{9}{14}$ **(D)** $\dfrac{5}{7}$ **(E)** $\dfrac{13}{18}$

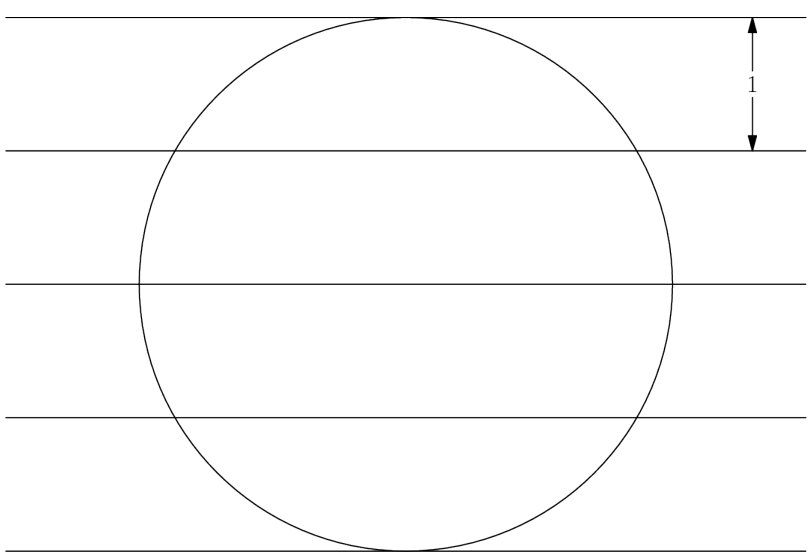
 # 21. A dinghy travels 90 miles. Its speed is x miles per hour for the first 30 miles, y miles per hour for the second 30 miles, and z miles per hour for the last 30 miles, where $x > y > z$. Suppose that

$$xy + yz + zx = 14\frac{1}{10} \quad \text{and} \quad xyz = 9.$$

How long did the journey take, in hours?

(A) 45 **(B)** 47 **(C)** $70\dfrac{1}{2}$ **(D)** 90 **(E)** 94

 # 22. Five equally-spaced parallel lines which are 1 unit apart cut a circle into four pieces. If two of the lines are tangent to the circle, what is the area of the smallest of the four pieces?



(A) $\frac{2\pi}{3} - \sqrt{3}$ **(B)** $\pi - \sqrt{3}$ **(C)** $\pi - 2$ **(D)** $\frac{4\pi}{3} - 2$ **(E)** $\frac{4\pi}{3} - \sqrt{3}$

 # 23. For how many positive integers n does $n^n - n^{n-1} + n^{n-2} - n^{n-1}$ have $3n-3$ factors?
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

 # 24. Let x be a positive multiple of 7 with exactly 30 factors. How many such x satisfy

$$\operatorname{lcm}(x, 30) = 5x?$$

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 10

 # 25. Maylin sticks together seven unit cubes to make the shape shown in figure 1. A plane $\mathcal{P}$ passes through all seven of the cubes. $\mathcal{P}$ passes through the midpoints of six of the edges of the center cube, as shown in figure 2.

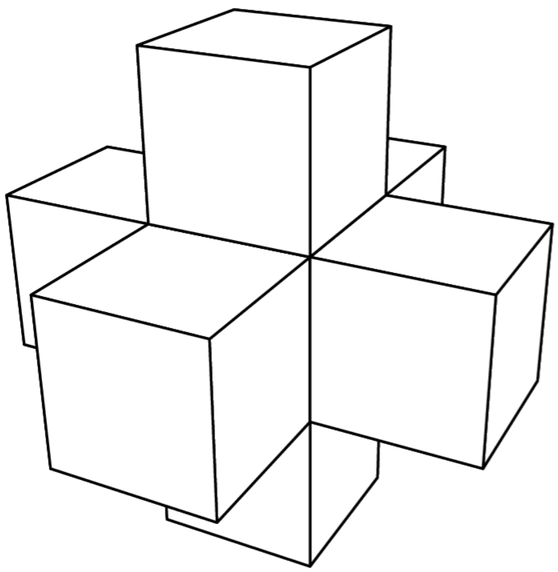


Figure 1

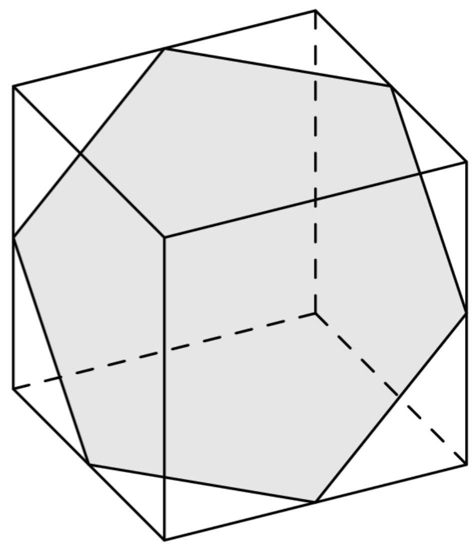


Figure 2

Let $\mathcal{R}$ be the area of the cross section the plane $\mathcal{P}$ makes with Maylin's shape. Find $\mathcal{R}^2$.
(**A**) $\dfrac{3}{16}$ (**B**) $\dfrac{27}{64}$ (**C**) $\dfrac{27}{16}$ (**D**) $\dfrac{27}{8}$ (**E**) $\dfrac{27}{4}$