# 2021 WMC 8

WMC Committee

This is the 2021 Winter Mathematics Competition for 8th graders and below, designed to imitate and provide practice for the MAA's AMC 8. The test was released on December 18, and submissions will be closed on January 15.

As with the MAA's AMC 8, you have 40 minutes to solve 25 multiple choice questions with answer choices A, B, C, D, and E. You will receive 1 point for every correct answer and 0 points for every answer left blank or incorrect. The only aids permitted are writing utensils, erasers, blank scratch paper, straightedge, and compass; in particular, no graph paper, calculators, or any other computing devices are allowed. And of course, do not cheat or discuss problems outside the designated private discussion forum while the contest is ongoing; this may result in disqualification.

The 2021 WMC 8 was made possible by the contributions of problem-writers and test-solvers **A1001**, **bissue**, **FalconMaster**, **mango5**, **nikenissan**, **peace09**, **Robin2**, and **tenebrine**. These problems aren't copyrighted or anything—we created this test to benefit the community!—but if you would like to use them, please cite accordingly (e.g. 2021 WMC 8 #1). Thanks!

1. The Gettysburg Address began with the famous phrase "four score and seven years ago". How many years is that? (Note that there are 20 years in a score.)

   (A) 27   **(B)** 47   **(C)** 67   **(D)** 87   **(E)** 107

2. What is the sum of the digits of the quotient $3,431,969,149 \div 7$?

   (A) 7   **(B)** 16   **(C)** 25   **(D)** 34   **(E)** 43

3. Five siblings share a pie that is divided into 12 slices. The three youngest siblings eat 4 slices altogether, while the three oldest siblings eat 9 slices altogether. How many slices does the middle sibling eat?

   (A) 1   **(B)** 2   **(C)** 3   **(D)** 4   **(E)** 5

4. Shown below is a student's sketch of the state of Nevada. Its western and northern borders are 30 millimeters long, and its eastern border is 60 millimeters long. What is the area of his sketch in square millimeters?
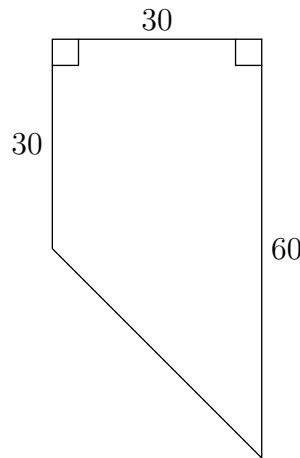
   (A) 600   **(B)** 900   **(C)** 1350   **(D)** 1800   **(E)** 2700

5. Chirag sells 2 pencils and 3 pens for 5 dollars and 8 pens and 13 pencils for 21 dollars. For how many dollars does he sell 34 pencils and 55 pens?

   (A) 56   **(B)** 67   **(C)** 78   **(D)** 89   **(E)** 100

6. At Valley Vince's Burgers, a burger costs $2.10. With Coupon A, Tyler can buy each burger for $2.00, and with Coupon B, his grand total is decreased by $1.80.

What is the least number of burgers he must buy in order for Coupon A to be a better deal than Coupon B?

(**A**) 18   (**B**) 19   (**C**) 20   (**D**) 21   (**E**) 22

7. An artist mixes red and blue paint to produce 100 pints of what she calls "perplexing purple". If she adds 10 pints of red paint and 10 pints of blue paint, the mixture becomes 20 percent red paint. How many pints of blue paint are there in 150 pints of perplexing purple?

(**A**) 129   (**B**) 135   (**C**) 141   (**D**) 147   (**E**) 153

8. Ashley divides a circular pizza into 4 slices so that each slice after the first is twice as large as the previous one. What is the degree measure of the central angle of the smallest slice?

(**A**) 15   (**B**) 18   (**C**) 20   (**D**) 24   (**E**) 30

9. Robin writes down the first N positive integers and removes all numbers with the digit 0. If the remaining list is 111 integers long, what is N?

(**A**) 123   (**B**) 132   (**C**) 133   (**D**) 134   (**E**) 143

10. Key, Fire, and Jupiter play a board game. Key plays first, then Fire, then Jupiter, then back to Key, and so on. At some point in the game, n turns have been played in total, and Fire has played 6 turns. What is the sum of all possible values of n?

(**A**) 18   (**B**) 35   (**C**) 36   (**D**) 37   (**E**) 54

11. A Connect Four grid has dimensions 6 by 7. A *four-in-a-row* is a set of 4 cells that are horizontally, vertically, or diagonally consecutive. How many four-in-a-rows are there in a Connect Four grid? Shown below is one possibility.
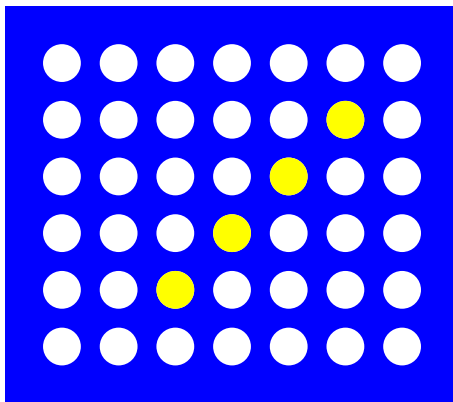
(A) 33 **(B)** 45 **(C)** 57 **(D)** 69 **(E)** 81

12. A building has dimensions 12 miles by 15 miles, and points X and Y are located 5 miles north and 8 miles east of the building respectively, as shown below. In a race from X to Y, Nick the ghost can drift through the building, while Harry the human must run around it, and both take optimal routes. To the nearest integer, how many miles shorter is Nick's route than Harry's?

X

5

15

12   8 Y
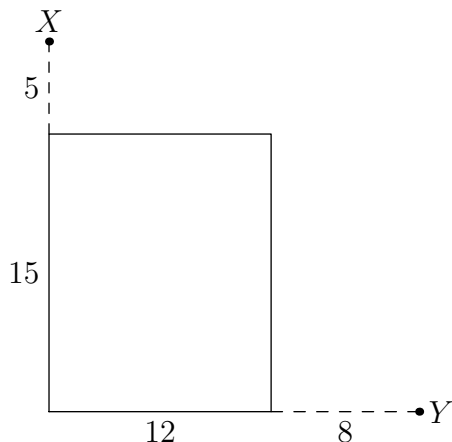
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

13. Ms. White assigns the numbers 1, 2, 3, 4, 5, and 6 to Alice, Bob, Carol, David, Eve, and Frederick in some order, so that each student knows their own number, but none of the others' numbers. She proceeds to announce the following:

- Alice's number is smaller than Bob's number,
- Carol's number is the square of David's number, and
- Eve's number is larger than Frederick's number.

Frederick promptly exclaims, "I know what Eve's number is!" What is the sum of Alice's Carol's, and Eve's numbers?

(A) 6 **(B)** 9 **(C)** 11 **(D)** 13 **(E)** 15

14. In the sport basketball, players can score 2-point shots, 3-point shots, or free throws (each worth 1 point). In a basketball game, a team scores a 2-point shots, b 3-point shots, and 17 free throws for a final score of 112. How many possible values are there for a?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

4

15. Two increasing sequences of positive integers both have first term 1. In one sequence, the mean of the first k terms is k, and in the other sequence, the range of the first k terms is k, for every positive integer $k > 1$. What is the difference between the 100th terms of the sequences?

(A) 98   (B) 99   (C) 100   (D) 101   (E) 102

16. Richard's unfair coin lands heads with four times the probability that it lands tails twice in a row. He will flip the coin once to see whether it lands heads or tails, and Vanessa, assuming the coin is fair, bets it will land tails. The probability that she wins the bet can be expressed in the form $\frac{\sqrt{a}-b}{c}$, where a, b, and c are positive integers and a is square-free. What is $a + b + c$?

(A) 20   (B) 23   (C) 26   (D) 29   (E) 32

17. Six circles are centered at the vertices of a regular hexagon, as shown below. Thirteen intersection points result, one at the center and six at the vertices of the hexagon. The remaining six points determine another hexagon with an area equalling how many times the area of the original hexagon?
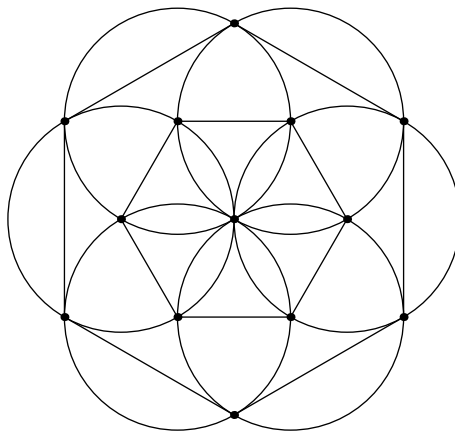

(A) 2   (B) $\frac{9}{4}$ (C) 3   (D) $\frac{16}{9}$ (E) 4

18. The ten digits from 0 to 9 are permuted and combined to form an integer. Two such integers are 0981276345 and 9018723654. What is the probability that this integer is a multiple of 2^1, 3^2, and 5^3?

(A) $\frac{1}{720}$ (B) $\frac{1}{360}$ (C) $\frac{1}{240}$ (D) $\frac{1}{180}$ (E) $\frac{1}{144}$

19. Sarah writes down the numbers 2 and 3 on a whiteboard. Every minute, she writes down the product of the previous two numbers. Hence, the next two numbers she

writes down are $2 \times 3 = 6$ and $3 \times 6 = 18$, and the fifth number is $6 \times 18 = 108$. How many divisors does the tenth number have?

(**A**) 770   (**B**) 910   (**C**) 1470   (**D**) 1610   (**E**) 1960

20. In how many ways can the 7 Harry Potter books and the 5 Percy Jackson books be displayed on a bookshelf so that both the Harry Potter books and the Percy Jackson books are in order? One such ordering is shown below.
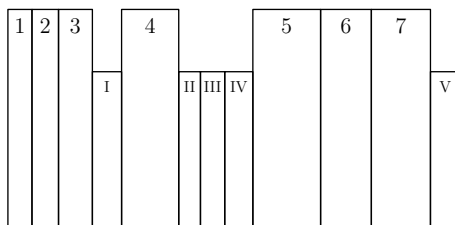

(**A**) 495   (**B**) 792   (**C**) 924   (**D**) 95040   (**E**) 3991680

21. A circle with integer area intersects a rectangle with side lengths 2 and 6 at 8 points. What is the sum of the smallest and largest possible areas of the circle?

(**A**) 56   (**B**) 57   (**C**) 58   (**D**) 59   (**E**) 60

22. Rohan labels the vertices of a 3 cm by 4 cm sheet of paper as P, Q, R, and S, in that order going clockwise, so that $\overline{PQ}$ is along the long side. Then, he folds the rectangle across $\overline{PR}$ as shown below to determine whether or not it is a line of symmetry. He finds it isn't, and proceeds to measure the length QS to see by how much the fold was "inaccurate". What length in centimeters should he obtain?
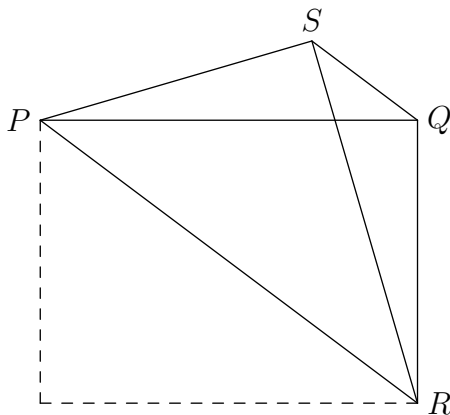
(**A**) 0.6   (**B**) 0.8   (**C**) 1   (**D**) 1.2   (**E**) 1.4

23. The sides and diagonals of convex octagon $ABCDEFGH$ are drawn. How many paths are there starting at A and ending at H from vertex to vertex along the drawn segments so that no vertex is visited more than once? The diagram below shows a valid path in green and an invalid path in red.
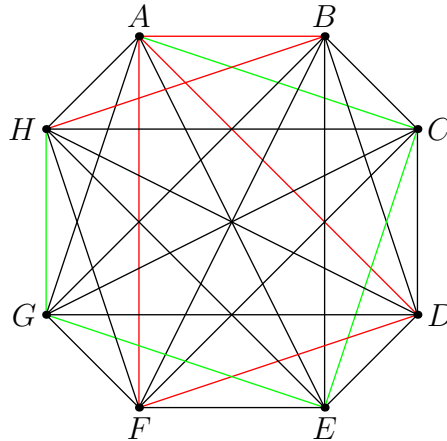
(A) 1797   (B) 1832   (C) 1877   (D) 1912   (E) 1957

24. A positive integer N is called *plentiful* if the product of its proper divisors (all divisors other than itself) is greater than N. For example, the number 12 is plentiful, because its proper divisors 1, 2, 3, 4, and 6 multiply to 144, which is greater than 12. How many of the first 50 positive integers are plentiful?

(A) 11   (B) 12   (C) 13   (D) 14   (E) 15

25. For any positive integer n, let $C(n)$ be the number of digits in n, and let $S(n)$ be the sum of its digits. How many positive integers n satisfy $C(n)^{C(n)} = S(n)$?

(A) 5   (B) 6   (C) 7   (D) 8   (E) 9