Ze Committee ZeMC

Ze Committee Ze Math Competitions

1st (maybe) annual

ZeMC 8

Saturday, December 25, 2021

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU TELL YOU.
2. This is a 25 question multiple choice test. For each question, only one answer choice is correct.
3. There is a Google Form to submit your answers; the link is found at https://forms.gle/NoM4fAAjqbuZqHun8.
   After you submit your form, you will immediately be able to view your results.
   Emails are not recorded with your response, but you will be asked for your AoPS username - this is to prevent trolls.
   You should receive a PM within a day of your submission containing a link to the private discussion form for this contest.
4. There is no penalty for guessing. Your score is the number of correct answers.
5. Only blank scratch paper, rulers, protractors, and erasers are allowed as aids. Calculators, grid paper and lined paper are NOT allowed. No problems on the test *require* the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, nobody will ask you to record your name and other information on the nonexistent answer sheet.
8. You will have 40 minutes to complete the test once you tell you to begin.
9. When you finish the exam, *sign your name* in the space provided at the bottom of the nonexistent answer sheet.

This is Bob. Say hi to Bob.

1. What is the average of all of the positive divisors of 15?

   (A) 4 **(B)** 5.5 **(C)** 6 **(D)** 7.5 **(E)** 8

2. New Zeland scored 3 more goals than Germany in the first half of the World Cup Finals, and Germany scored 1 more goal than New Zeland in the second half. If Germany scored 4 goals in total, how many goals did New Zeland score?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Deniz runs at a constant rate of 10 miles per hour, and Niall runs at a constant rate of 8 miles per hour. They both start running from the same point. How many miles will Deniz have run when he is one mile ahead of Niall?

   (A) 4 **(B)** 4.5 **(C)** 5 **(D)** 5.5 **(E)** 6

4. A semicircle has a perimeter of $8 + 4\pi$ units. What is its area, in square units?

   (A) 4π **(B)** 8π **(C)** 16π **(D)** 32π **(E)** 64π

5. There are 4 pencils in a box, 8 pencils in a package and 3 packages in a container. Cruise and Wes have the same number of pencils. Cruise has one container worth of pencils. Wes has n boxes worth of pencils. What is n?

   (A) 6 **(B)** 12 **(C)** 24 **(D)** 48 **(E)** 96

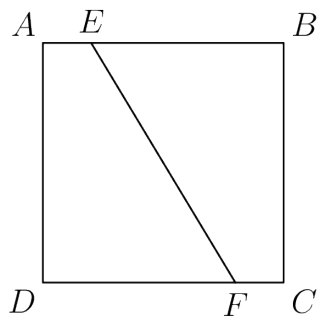
6. The angles of a quadrilateral are distinct positive integers. What is the largest possible degree measure of an angle in this quadrilateral?

   (A) 353 **(B)** 354 **(C)** 355 **(D)** 356 **(E)** 357

7. The mean, median and unique mode of four numbers are 4, 3 and 2, respectively. What is the range of these four numbers?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. Square $ABCD$ has a side length of 8. E and F are points on $\overline{AB}$ and $\overline{DC}$, respectively, such that $EF = 10$. If $AE = FC = x$, what is x?



(A) $\frac{1}{2}$ **(B)** 1 **(C)** $\frac{3}{2}$ **(D)** 2 **(E)** $\frac{5}{2}$

9. The front wheel of a bicycle has an **area** of π square feet, and the back wheel has a **circumference** of π feet. When the bicycle is moving, for every revolution that the front wheel makes, how many revolutions does the back wheel make?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

10. One of textbook A and one of textbook B are stacked on top of each other, and the weight of the stack is 25 ounces. Two of textbook C are added to the stack, and the weight of the stack increases to 55 ounces. Finally, one of textbook C and three of textbook A are added to the stack, and the weight of the stack increases to 103 ounces. How many ounces does textbook B weigh?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

11. Tinson evaluates the expression

$$\left(\frac{a}{4}\right)^b \cdot \left(\frac{b}{14}\right)^a$$

by multiplying both the numerators and denominators separately. His result is a fraction that is already simplified, with a value less than 1. If a and b are positive integers with $a > b$, find the smallest possible value of $a + b$.

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

12. Let $f(x)$ equal the largest even integer less or equal to x. If x is a randomly chosen number from -5 to 5, with uniform distribution, what is the probability that $f(x) > 0$? Note that x does not have to be an integer.

(A) $\frac{3}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{7}{10}$

13. What is $\frac{1}{2} \cdot \left(-\frac{2}{3}\right) \cdot \frac{3}{4} \cdot \left(-\frac{4}{5}\right) \cdot \ldots \cdot \left(-\frac{100}{101}\right)$?

(A) -1 **(B)** $-\frac{100}{101}$ **(C)** $-\frac{1}{101}$ **(D)** $\frac{1}{101}$ **(E)** $\frac{100}{101}$

14. Adi has twice as many quarters as dimes, twice as many dimes as nickels and twice as many nickels as pennies. If his coins are worth a positive integer value of dollars, what is the fewest number of coins he could have?

(A) 300 **(B)** 450 **(C)** 800 **(D)** 1500 **(E)** 1550

15. AJ is thinking of a positive integer. He makes the following claims:

- My number is odd.
- My number contains the digit 7.
- My number is a perfect square.
- My number is a perfect cube.
- My number is greater than 1 and less than 30.

At most how many of these following claims can be simultaneously true?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

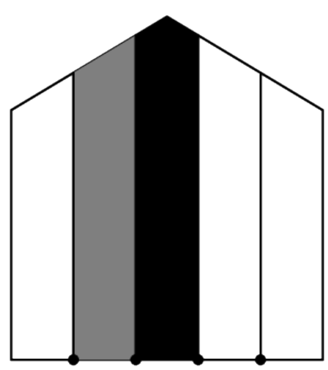
16. Mr. Danahey is returning graded tests back to his 20 (distinguishable) students - each test belongs to exactly 1 student. If exactly three of them receive a test that is not theirs, how many ways could Mr. Danahey have distributed the tests?

(A) 190 **(B)** 380 **(C)** 1140 **(D)** 1710 **(E)** 2280

17. The quotient of two positive integers is equal to an integer divisible by 9. The sum of the same two positive integers is equal to an integer divisible by 3. What is the largest integer that will always divide their product?

(A) 3 **(B)** 9 **(C)** 27 **(D)** 81 **(E)** 243

18. Pentagon $ABCDE$ satisfies $\angle A = \angle B = 90^\circ$ and $ED = DC$. Four points are placed, equally spaced, along AB, and perpendiculars are drawn through each point, as shown below. If the darkly shaded region has an area of 13 square units and the lightly shaded region has an area of 10 square units, what is the area of the entire pentagon, in square units?



(A) 45 **(B)** 47 **(C)** 49 **(D)** 58 **(E)** 60

19. Rayan randomly chooses a positive six digit integer, with uniform distribution. He rewrites the integer, but if a block of adjacent digits have the same value, he only writes the first digit down. For example, if Rayan chooses 222422, he writes down 242. What is the expected number of digits he will need to write down?

(A) $\frac{419}{100}$ **(B)** $\frac{2319}{500}$ **(C)** 5 **(D)** $\frac{11}{2}$ **(E)** $\frac{27}{5}$

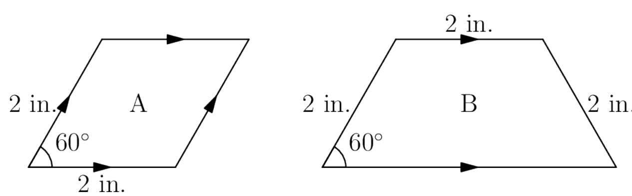
20. On a 4 by 4 checkers board, how many ways are there to place 7 indistinguishable chips on distinct squares of the board such that no two occupied squares share an edge?

(A) 16 **(B)** 20 **(C)** 24 **(D)** 28 **(E)** 32

21. The current time displayed on an analog clock is 12 : 05. In the next 20 minutes, on how many occasions will the angle, in degrees, formed between the hour and minute hand, be an integer? Assume that both hands move at a constant rate (i.e. their movement is continuous).

(A) 19 **(B)** 39 **(C)** 78 **(D)** 110 **(E)** 138

22. Sophie has 21 pieces of cut-out paper - 9 with the dimensions of rhombus A, and 12 with the dimensions of trapezoid B. She arranges the pieces together, such that no two pieces overlap, to form a regular hexagon with no holes inside. Flipping pieces of paper over is allowed. In inches, what is the maximum possible perimeter of this hexagon?



(A) 24 **(B)** 36 **(C)** 48 **(D)** 60 **(E)** 72

23. Let n be a positive number such that $\frac{72!}{n}$ is a multiple of 2^n, but not a multiple of 2^{n+1}. What is the sum of all possible values of n?

(A) 119 **(B)** 122 **(C)** 128 **(D)** 129 **(E)** 132

24. Let a, b, c and d be randomly chosen integers between 1 and 10, inclusive. Let $f(x) = ax + b$ and $g(x) = cx+d$. Given that $f(x) \neq g(x)$ when $x \neq 0$, what is the probability that $f(0) = g(0)$?

(A) $\frac{9}{20}$ **(B)** $\frac{9}{19}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{10}{19}$ **(E)** $\frac{11}{20}$

25. Let $\triangle ABC$ be a triangle such that $\angle B = 90$, and $AB = BC$. P is a point on the same plane, satisfying $BP = 2$ and $AP = CP = 3$. If X is the area of $\triangle ABC$, what is the sum of all possible values of X?

(A) $9 - 2\sqrt{14}$ **(B)** 5 **(C)** $2\sqrt{14}$ **(D)** 8 **(E)** 9

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DO NOT OPEN UNTIL SATURDAY, December 25, 2021

Administration on an earlier date will disqualify you from getting a cookie.

- All the information needed to administer this competition is contained in the nonexistent AMC8 Teacher's Manual. PLEASE DO NOT READ THE MANUAL AS IT CONTAINS DEADLY RADIATION INSIDE.
- Answer sheets must be returned to the Ze Committee ZeMC office within 3 seconds of the competition administration. Use an overnight or 2-day shipping service, with a tracking number, to guarantee the timely arrival of these answer sheets. If you wish for the answer sheets to get tossed into the dumpster, USPS overnight is strongly recommended.
- The publication, reproduction or unproduction of the problems on this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email or friends (if you have them) during this period is a violation of competition rules.

The ZeMC 8 is made possible by the contributions of the following problem-writers and test-solvers:

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