

1. $(4 * (-4)^2)((-2)^{-4}) = 64 * \frac{1}{16} = 4.$
2. Working backwards, we find the amount he has on the day of the shopping trip to be $((12 + 2) * 3) + 5) * 2 = 94.$ Since this is $\frac{1}{10}$ of the total salary, our answer is 940.
3. If $n = 3,$ then 6,7,8 satisfy the given conditions. If $n > 3,$ then the mean would be higher, so $n = 3$ is the answer.
4. Using Vieta's Formulas, our answer is simply $\frac{-b}{a} = -4.$
5. We have $9^n + n = \frac{7}{2}.$ Testing the answer choices, we find that $\frac{1}{2}$ works.
6. Listing out the possible values for the answer we have $1^3 + 2^3, 1^3 + 3^3, 1^3 + 4^3, 2^3 + 3^3, 2^3 + 4^3, 3^3 + 4^3.$ These are 6 possibilities.
7. There are 16 options for the first face card out of 78, so start off with $\frac{16}{78}.$ Then for the second one, you cannot use any of the face cards with the same suit as the first one, so you only have 16-4=12 options out of 77. So multiply $\frac{16}{78}$ by $\frac{12}{77}$ and simplify to get $\frac{32}{1001}$.
8. The point starts at (0,3). Then it goes to (3,0) after the 90 degree rotation. Once reflected across the y axis, it's at (-3,0). When rotated a half turn about point (2,1) (a 180 degree rotation) this means that (2,1) is the midpoint between (-3,0) and the new point, so the new point is (7,2). Then, when reflected across the line x=y, you switch x and y to get the final point, (2,7).
9. Since DFE is the medial triangle of right triangle ABC, it cuts it into 4 triangles of equal area. Therefore the answer is $\frac{3}{4}$ of the area of ABC, which is $(\frac{3}{4})(\frac{1}{2})(14)(6) = 31.5.$
10. For the equation to have a solution when $x = 3,$ we must have $k(3 - a) = 0.$ However, since $a \neq 3,$ we must have $k = 0.$ Therefore the answer is 1 possible value of k.
11. If $p^3 \equiv 2 \pmod{3},$ then $p \equiv 2 \pmod{3}.$ All of the primes less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19 and out of these, 2, 5, 11, and 17 can be $p,$ so the probability is $\frac{4}{8},$ or $\frac{1}{2}.$

12. $x = \frac{2^{124}-1}{2^{62}-1} = 2^{62} + 1 \Rightarrow 4x - 5 = 2^{64} - 1$. This factors as $(2^{32} + 1)(2^{16} + 1)(2^8 + 1)(2^4 + 1)(2^2 + 1)(2 + 1)(2 - 1)$. The smallest prime factors of this are 3, 5, and 17.
13. Again, using Vieta, $\frac{-b}{a} = 12$.
14. It cannot be determined, because 2 sides do not define a right triangle.
15. It is easy to see that there are 64 1x1 cubes at the end. From these, we need to subtract the ones that are only blue and the ones that are only red. All of the cubes on the inside are only blue, and there are $2^3 = 8$ of them and then all of the cubes on the corners are only red as they weren't "exposed" to blue after the first cuts. So 8 are only red. Then, you have $64-8-8=48$.
16. $\sqrt{\sqrt[3]{6 \cdot 24 \cdot 96} + 1} = \sqrt{\sqrt[3]{2^9 \cdot 3^3} + 1} = \sqrt{2^3 \cdot 3 + 1} = \sqrt{24 + 1} = \sqrt{25} = 5$.
17. $2^5 3^4 4^3 5^9 = 2^{11} 3^4 5^9 = 10^9 * 324$ writing in scientific notation $10^{11} * 3.24$ so $11 + \sqrt{324} = 11+18 = 29$.
18. Say Ally and Brian got x candies while the third kid got y and the fourth kid got z . Then we have that $2x+y+z=8$ where $x,y,z>0$, So Let $x=k+1, y=m+1, z=n+1$ where m,n,k are all nonnegative. Then $2k+m+n=4$ and we need to find the number of possible solutions. Now there may be an easier way to solve this but we have 3 cases where $k=0,1,2$. If $k=0$, $m+n=4$ and there are 5 solutions, if $k=1, m+n=2$ and there are 3 solutions, and if $k=2, m+n=0$ and there is 1 solution, so $5+3+1=9$.
19. Let x,y be the sides of the rectangle. We want to find $\sqrt{x^2 + y^2}$. We have $x + y = 17$ and $xy = 42$. $(x + y)^2 - 2xy = x^2 + y^2 = 205$, so the answer is $\sqrt{205}$.
20. Solution 1: First note that counting cases would be a lot of work and 6 is very close to 7. We do casework on the results of flipping Coin A and Coin B 6 times. We can either # heads of A > # heads of B or # heads of A=#heads of B and A gets a head on the 7th toss.

For the first case, the probability is $\frac{(1-\frac{\binom{6}{3}}{64})}{2} = \frac{11}{32}$. For the second case, the probability is $\frac{\frac{\binom{6}{3}}{64}}{2} = \frac{5}{32}$. Thus, the total probability is $\frac{11}{32} + \frac{5}{32} = \frac{1}{2}$.

Note: This was my favourite problem on this mock test ~Tan :)

Solution 2: Let's consider coin A's first 6 flips and coin B's first 6 flips. At this point, we expect these values to be equal. If coin A flips tails, then the condition is not satisfied. If coin A flips heads, then the condition is satisfied. So, our answer is $\frac{1}{2}$.

21. At 10:00, the distance between Austin and Jessica is 130 km as Austin traveled 20 km by then (it's been an hour). So then at 10:30, the train passes Austin and at this point, he's 10 km away from where he was at 10:00. Then at 11:15, the train passes Jessica and by then it had been 75 minutes so she was 75 km away from her starting point, making her $130-75=55$ km away from where Austin was at 10:00. This also makes her $55-10=45$ km away from where the train was at 10:30. Then, the train traveled 45 km from 10:30 to 11:15 (in 45 minutes), so it was going at 60 km/h.

22. Let $X = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2016}$ and let $Y = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2015}$. Note that $X = Y + \frac{1}{2016}$. Our expression now becomes $Y(X - 1) - X(Y - 1) = XY - Y - XY + X = X - Y = \frac{1}{2016}$.

23. Label A, C, and E as "even" and B, D, and F as "odd." Note that every move must take the bug from even to odd or vice versa. This means that Alan must end on an "even" vertex. Trying possibilities yields that all 3 cases are possible, so our answer is 3.

24. If this is true, then either the exponent is 0, the base is 1, or the base is -1 while the exponent is even. In order for the exponent to be 0, $x^2 - 5x + 2 = 0$ and by Vieta's, the sum of the roots for this is 5. Then, if the base is 1, $x^2 - 4x + 1 = 0$ and here, the sum of the roots is 4. Note that these don't have the same roots but if they did, I'd actually have to find the roots to find their sum without counting any twice. Then, if the base is -1, $x^2 - 4x + 3 = 0$ and this time I actually have to find the roots so I can plug them in to the exponent and see that it's even. So the roots are 3 and 1 and the exponent is then -4 and -2, which are even so both work. So the sum of all the roots are $5+4+4=13$.

25. So our answer is xn (the number of teams) times $xn - n$ (number of teams they each have to play against, times 2 and divided by 2. This is equivalent to answer choice B.