lethan3's Mock AMC 8 Answers and Solutions

Answer Key

BBAEE/BCCEB/EDCDD/ACDBD/DCCDD

Solutions

1. Notice that trees and vines are not fruits. Therefore we sum $3+15=\boxed{\text{(B)}18}$.

2. 12 toy planes are equivalent to 21 toy trucks for Bob. Therefore, if we divide both quantities by 4, they will still be equivalent to Bob. Therefore 3 toy planes are equivalent to $5.25\approx\boxed{\text{(B)}5}$ toy trucks to Bob.

3. We can use algebra to solve this problem. Let a by Abby's age and b be Bryan's age. (1) $a+b=121$, and $|a-b|=43$. However, we know from Edgar that Bryan is older than Abby, so (2) $b-a=43$. Adding equations (1) and (2) gives us $2b=164$ and therefore $b=\boxed{\text{(A)}82}$.

4. Let one of the points be (x,y). The other point is the reflection of this point over the line $y=12$, which means the other point is $(x,24-y)$. The sum of the y-coordinates is 24, while the difference of the x-coordinates is 0. Therefore the answer is $\boxed{\text{(E)}24}$.

5. We write a closed form for the amount each mathematician eats instead of just summing all of the numbers. If there was a 0th mathematician, he would eat 8 scoops. Since the nth mathematician eats $(1/2)^n$ times the zeroth mathematician, the closed form is $8(1/2)^n$. Therefore the last mathematician eats $1/32$ scoops of ice-cream. Notice that the denominator of the answer must match the denominator of the number of scoops the last person ate for $n\geq 4$. Therefore the answer is $\boxed{\text{(E)}7\frac{31}{32}}$.

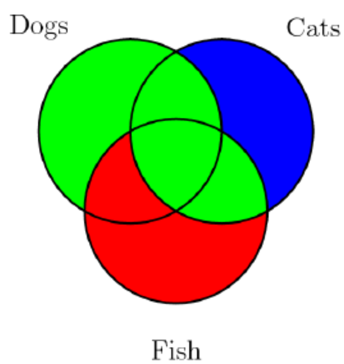
6. Let a be the number of square faces and b be the number of triangular faces. Then knowing there are 38 faces, $a+b=38$. If there are 60 edges, then each edge actually corresponds to two sides for each of the faces it borders. So $4a+3b=120$, and solving gives $a=6$ and $b=32$. Therefore the answer is $\boxed{\text{(B)}19}$.

7. Since the overlaps of the circles are counted twice, we can simply sum the areas of all of the small circles to get the desired area. Since each small circle has $1/2$ the radius and therefore $1/4$ the area of the big circle, the wanted area and the big circle's area are equal hence $\boxed{(C)1}$.

8. Note that $f(n) = f(-n)$. Therefore, if $g(f(n)) = 1$, then $g(f(-n)) = 1$. If any positive integer n satisfies this property, its negative does also. Therefore the sum of all possible values of n is 0. Hence the answer is $24/2 = \boxed{(C)12}$.

9. At first glance, it seems that the ant can do 5 possible actions in one step: go up, down, left, right, or stay still giving an answer of $5^3 = 125$. However, this is not the case as the ant can follow the hypotenuse of a 3-4-5 right triangle. Since there are 8 ways to do this, the answer is $(5+8)^3 = \boxed{(E)2197}$.

10. To simplify the problem, we first note that the amount of students that have pets is 50. Now, we can draw a Venn diagram.



Note that the first group of students ("21 students have fish but no cats") is the red portion and the second group of students ("19 students either have dogs and no fish, or cats and fish") is the green portion. What we are trying to find (the number of students that only have cats) is the blue portion. Note that this is just all the students that have pets, minus the red and green regions. Therefore, the answer is $50 - 21 - 19 = \boxed{(B)10}$.

11. We can make an equation and solve it. For the sake of simplicity, let their normal speed be 1 m/s. Then normally they would take 100 seconds to finish the race. However, Czerny takes 83/80 that amount, or 415/4 seconds. Then, $n + 5/4(100 - n) = 415/4$, since after slipping, Czerny takes 5/4 the time since he moves at 4/5 the rate. Solving gives $\boxed{(E)85}$.

12. Note that diagonals AE and BD are parallel, and CG and DF are parallel as well. Therefore, the quadrilateral is a parallelogram, and the degree measure we want is $\frac{180}{1}$ giving us $\boxed{\text{(D)}181}$.

13. Looking at option C) (blue, blue, green), note that a blue face is opposite the green face; therefore they can't share a vertex hence $\boxed{\text{(C) blue, blue, green}}$.

14. By linearity of expectation, the expected value of Kayla's roll is twice the value than if she rolled only one die. The expected value from one die is $\frac{3 \cdot 1 + 1 \cdot 2 + 2 \cdot 3}{6} = \frac{11}{6}$. Therefore, from two dice her expected value is $\boxed{\text{(D)}\frac{11}{3}}$.

15. First, notice that the yoga ball actually has a weight of 12 ounces, not 0.75 ounces. Now, multiplying all the numbers on Bob's ball (while squaring the distance dropped) gives us 80, which corresponds proportionally to 0.5 cm squish. Now, multiplying all the numbers on Wesley's ball (while squaring the distance dropped) gives us 2400 (noting the change in units). Notice that for Bob, we divide by 160 to get the squish. We do the same thing to 2400 to get $\boxed{\text{(D)}15}$.

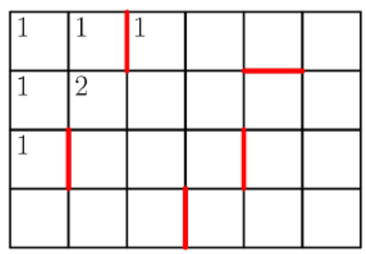
16. The figure created by the spinning die consists of two cones with their bases coinciding. The height of one of these cones is half the height of the octahedron, which is $\sqrt{2}$ the side length of the octahedron (due to the sides forming a square in the octahedron). Therefore the height of one of the cones is $\frac{\sqrt{2}}{2}$. The radius of the cone is the same as the height of the cone since both are the distance from the center of the octahedron to one of its vertices. Therefore the volume of one of the cones is $\frac{2\sqrt{2}\pi}{3 \cdot 2^3}$ and the volume of both is $\frac{2\sqrt{2}\pi}{3 \cdot 4} = \frac{\pi\sqrt{2}}{6}$ and the answer is $\boxed{\text{(A)}8}$.

17. If there were no restrictions, Gggxyz could color the grid in 2^{27} ways. However, she can only color an odd number of cubes. Notice that coloring an even number of cubes is the same thing as not coloring an odd number of cubes, which has the same number of possibilities as coloring an odd number of cubes. Therefore, the number of ways to color an even number of

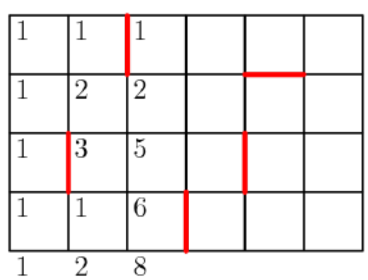
cubes is the same as the number of ways to color an odd number of cubes, and this is $2^{27}/2 = 2^{26}$ and the answer is $\boxed{\text{(C)}26}$.

18. Notice that the new polyhedron is formed by removing triangular pyramids at the vertices of the octahedron, and that the triangular pyramids each have half the side lengths of the octahedron. Now, consider a pair of these triangular pyramids that are cut off from opposite vertices. Since the octahedron can be cut into half along the 4 edges that separate these two vertices, these two pyramids collectively cut off 1/8 or the whole octahedron. Since there are 3 pairs in total, they cut off 3/8 of the whole octahedron collectively. The resulting polyhedron has 5/8 of the volume of the octahedron, and the answer is $\boxed{\text{(D)}5/8}$.

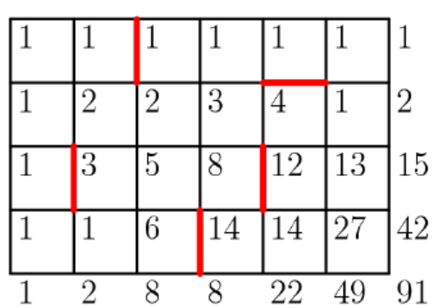
19. We label every lattice point in the grid based on how many ways Ness can get to it. Every point's label should be the sum of the labels directly above and to the left of it, except if directly above the point or to the left of the point is a red line segment. This is how we start:



Now, there is no possible way to get from (2,4) to (2,3), so to label (2,3), we only use (1,3). Continuing in this fashion gives:

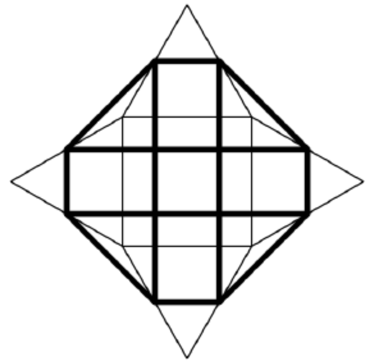


...and so forth...



We see that our answer is therefore $\boxed{\text{(B)}91}$.

20. We divide the octagon as shown:



Each of the rectangles has width 1 and length $1/2 + \sqrt{3}/2$, so each rectangle has area $\frac{1+\sqrt{3}}{2}$ and the total area comprised of rectangles is $2 + 2\sqrt{3}$.

Each of the isosceles right triangles have legs of $1/2 + \sqrt{3}/2$, so the total area of right triangles is $2 + \sqrt{3}$.

The square in the center has side length 1, so its area is 1.

Summing up all the areas gives $5 + 3\sqrt{3}$, so the answer is $\boxed{\text{(D)}11}$.

21. Let x be the number of chocolate bars Charlie has. We know that x has to be 3 more than a multiple of 7. Now we try to find a value of x that is also 5 more than a multiple of 6. Listing out numbers that are 3 more than a multiple of 7, we have:

3, 10, 17

So now our number is 17 more than a multiple of 42. Now we try to find a number that is 3 more than a multiple of 5:

17, 59, 101, 143

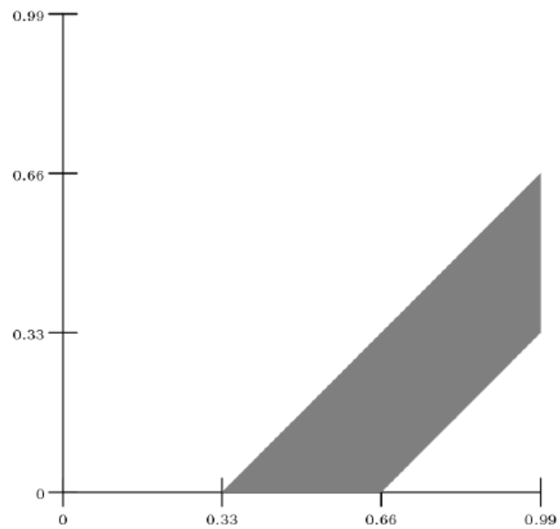
Now we know our number is 143 more than a multiple of 210. Trying to find the largest such number less than 1000, we have:

143, 353, 563, 773, 983

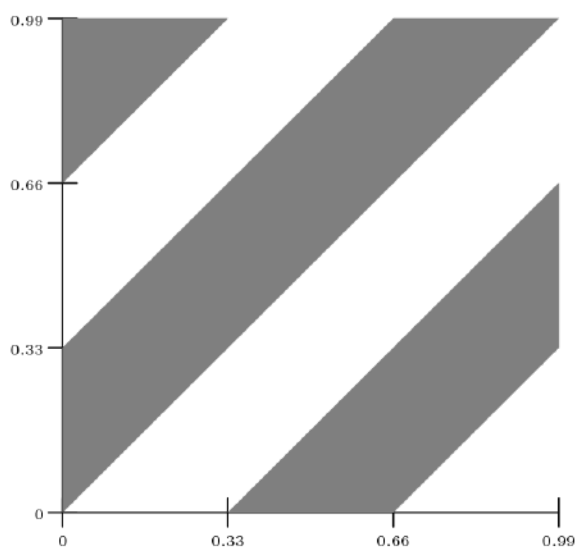
The next number will go over the limit of 1000. Therefore, the sum of the digits is $\boxed{\text{(D)}20}$.

22. We use geometric probability. After each result, we will graph the resulting region.

If $\lfloor 3|y-x+1| \rfloor = 1$, then $3|y-x+1|$ is in the range $[1, 2)$ and $|y-x+1|$ is in the range $[1/3, 2/3)$. Therefore, $y-x+1$ is either in $(-2/3, -1/3]$ or $[1/3, 2/3)$ and $y-x$ is either in $(-5/3, -4/3]$ (which is not possible) or in $[-2/3, -1/3)$. This means that the graph is in the shaded region shown: (we don't have to worry about boundaries because they don't matter in geometric probability)

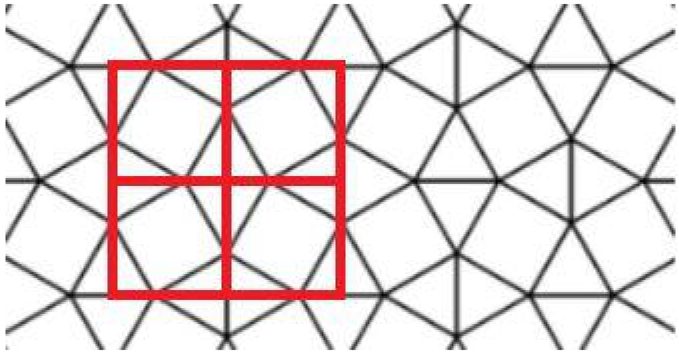


Repeating for the rest of the set gives:



Clearly, $\boxed{\text{(C)}\frac{1}{2}}$ is the answer.

23. Notice that we can break up the tessellation into squares:



If each polygon has a side length of 1, each red square has a side length of $\frac{1+\sqrt{3}}{2}$, and each black square has a side length of 1. Therefore, each black square occupies $\frac{4+2\sqrt{3}}{1}$ of each red square, and the answer is $4+2+3+1=\boxed{\text{(C)}10}$.

24. Our equation is $xyz-9xy-4xz-5yz+36x+45y+20z-210=0$, which we hope can be factored into the form $(x-a)(y-b)(z-c)=n$. Factoring this looks like a mess, until we notice that the coefficients of xy, xz, and yz are $-c$, $-b$, and $-a$, respectively. Therefore, $a=5$, $b=4$, and $c=9$. The other coefficients all match except for the constant term, so we get the equation $(x-5)(y-4)(z-9)=30$.

Now we need to find how many ways are there to split 30 into 3 factors. Listing them out according to how many 1's there are gives 2, 3, 5; 1, 2, 15; 1, 3, 10; 1, 5, 6; and 1, 1, 30.

For a set of these factors d, e, and f that we assign to $x-5$, $y-4$, and $z-9$ respectively, we have the set of solutions $\{x=d+5, y=e+4, z=f+9\}$, $\{x=-d+5, y=-e+4, z=f+9\}$, $\{x=d+5, y=-e+4, z=-f+9\}$, and $\{x=-d+5, y=e+4, z=-f+9\}$. When we sum these all up, we get $18\cdot 4=72$ since all of the variables cancel out. Now we know that for every permutation of factors, 72 gets added to the total.

There are $6\cdot 4+3=27$ possible permutations, from the second paragraph. Therefore the answer is $27\cdot 72=\boxed{\text{(D)}1944}$.

25. Our original series is $S = \frac{1}{2} + \frac{8}{4} + \frac{27}{8} + \frac{64}{16} + \frac{125}{32} + \frac{216}{64}...$. Therefore $\frac{S}{2} = \frac{1}{4} + \frac{8}{8} + \frac{27}{16} + \frac{64}{32} + \frac{125}{64}...$ and we can subtract this from S by matching denominators (such that 1/4 is subtracted from 8/4, 8/8 is subtracted from 27/8, and so forth) to get a result of $\frac{S}{2} = \frac{1}{2} + \frac{7}{4} + \frac{19}{8} + \frac{37}{16} + \frac{61}{32} + \frac{91}{64}...$.

Now $\frac{S}{4} = \frac{1}{4} + \frac{7}{8} + \frac{19}{16} + \frac{37}{32} + \frac{61}{64}...$ and we can subtract like before to get $\frac{S}{4} = \frac{1}{2} + \frac{6}{4} + \frac{12}{8} + \frac{18}{16} + \frac{24}{32} + \frac{30}{64}...$. Note that if we subtract 1/2 from each side, this becomes an arithmetico-geometric sequence. We do that to get $\frac{S}{4} - \frac{1}{2} = \frac{6}{4} + \frac{12}{8} + \frac{18}{16} + \frac{24}{32} + \frac{30}{64}...$ so $\frac{S}{4} - \frac{1}{2} = \frac{3}{2} + \frac{6}{4} + \frac{9}{8} + \frac{12}{16} + \frac{15}{32}...$

We repeat again, giving us $\frac{S}{8} - \frac{1}{4} = \frac{3}{2} + \frac{3}{4} + \frac{3}{8}...$, so $\frac{S}{8} - \frac{1}{4} = 3$. Therefore, $\frac{S}{8} = \frac{13}{4}$ and $S = 26$. The answer is $\boxed{\text{(D)}26}$.