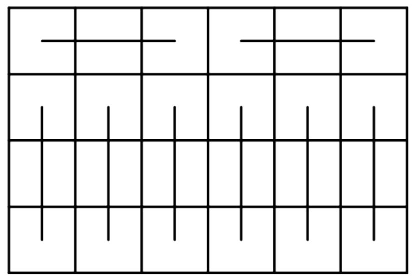
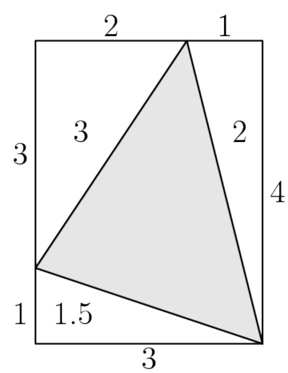
 # 1. Answer: (D). Note that $2020 \times 10 = 20200$, so $2020 \times 10 + 2020 = 20200 + 2020 = 22220$.

 # 2. Answer: (B). There are three squares in the rectangle and 24 squares in the grid, so we can make at most $\frac{24}{3} = 8$ rectangles. The picture below shows this is possible.



 # 3. Answer: (B). The sum is 1111110, which has one zero.

 # 4. Answer: (B). The area of the rectangle is 12, and the areas of the white triangles are 1.5, 3, and 4. Thus the area of the shaded triangle is $12 - 1.5 - 3 - 2 = 5.5$.



 # 5. Answer: (C). Each rotation corresponds to riding 108 centimeters, so riding a kilometer is riding $100,000$ centimeters. Therefore the answer is $\frac{100,000}{108} \approx \frac{100,000}{100} = 1000$.

 # 6. Answer: (A). Note that the questions are missing the numbers 1, 2, 3, 4, and 5, respectively. Thus Bartek cannot have four cards, because then he must have answered 3 to at least one question. So Bartek must have five cards. Since he answered 4 to every question, one of the cards he has is missing. Thus, he has the cards 1, 2, 3, 4, and 5, which sum to 15.

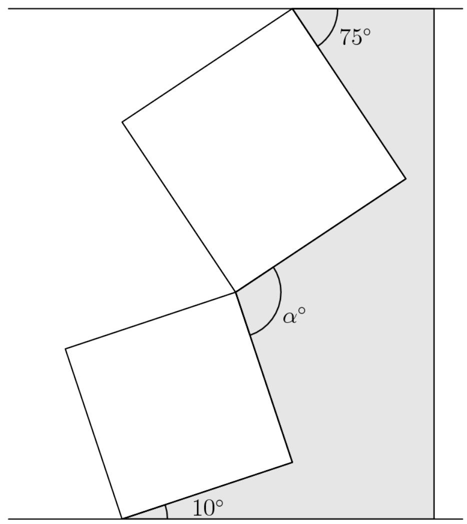
 # 7. Answer: (C). Raising the first equality to the fourth power gives $a^8 = b^{12}$. Raising the second equality to the third power gives $b^{12} = c^{15}$. Therefore $a^8 = c^{15}$. Squaring both sides gives $a^{16} = c^{30}$.

 # 8. Answer: (D). The sum of the numbers originally is $90n$. After $90n$ is added, the sum is $180n$. Thus the average is $\frac{180n}{n+1}$, which is equal to 162. Thus $\frac{180n}{n+1} = 162$ or $180n = 162n + 162$, so $n = 14$.

 # 9. Answer: (B). Draw a line segment perpendicular to both parallel lines, as shown. Now the angles of the shaded heptagon are, in clockwise order,

$$90°, 90°, 10°, 270°, \alpha°, 270°, 75°.$$

The sum of the angles in a septagon is $180° \times (7-2) = 900°$. Solving yields $\alpha = 95°$.



 # 10. Answer: (A). Cancel $(x-3)$ from the numerator and denominator, noting $x = 3$ is not a solution. Then we have $\frac{(x-2)}{(x-4)} = -1$ or $x - 2 = 4 - x$, so $x = 3$. But $x = 3$ is not a solution, as we have seen. Thus there are no real solutions.

 # 11. Answer: (D). Let x_1 be the area of $\mathcal{T}_1$ and let x_2 be the area of $\mathcal{T}_2$. Then $x_1 - 10 = 3(x_2 - 10)$ and $x_1 = 2x_2$. Solving for x_1 yields $x_1 = 40$.

 # 12. Answer: (C). Since there are four prime digits, the number must have at most four digits. Now do casework:

One digit: 4 numbers; two digits: $4 \cdot 3 = 12$ numbers; three digits: $4 \cdot 3 \cdot 2 = 24$ numbers; four digits: $4 \cdot 3 \cdot 2 \cdot 1 = 24$ numbers. Thus there are a total of $4 + 12 + 24 + 24 = 64$ such numbers.

 # 13. Answer: (A). The only ways to split three apples among four students are $3+0+0+0$, $2+1+0+0$, and $1+1+1+0$. There are four ways to arrange the numbers $(3,0,0,0)$, twelve ways to arrange the numbers $(2,1,0,0)$, and four ways to arrange the numbers $(1,1,1,0)$. Hence the desired probability is $\frac{4}{4+12+4} = \frac{1}{5}$.

 # 14. Answer: (A). The product $A \cdot B$ is equal to

$$\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{5}+\sqrt{4}\right)\left(\sqrt{5}-\sqrt{4}\right)\cdots$$
$$\cdots\left(\sqrt{2021}+\sqrt{2020}\right)\left(\sqrt{2021}-\sqrt{2020}\right),$$

which by the difference-of-squares formula equals

$$(3-2)(4-5)\cdots(2021-2020) = 1 \cdot 1 \cdots 1 = 1.$$

 # 15. Answer: (A). We have $\frac{20^{\left(20^{20}\right)}}{(20^{20})^{20}} = \frac{20^{\left(20^{20}\right)}}{20^{400}} = 20^{20^{20}-400} = 20^n$, so $n = 20^{20} - 400 = 20^2\left(20^{18} - 1\right)$. Clearly $20^{18} - 1$ has no factors of 20, so the largest power of 20 that divides n is 20^2.

 # 16. Answer: (A). Consider the two types of circles individually. Note that each great meridian splits the surface of the globe into two parts. Thus, n great meridians split the globe into $2n$ parts. Now note that n parallels split the globe into $n+1$ parts. Thus 12 parallels and 12 meridians split the globe into $24 \cdot 13 = 312$ parts.

Now we'll find the probability asked for. Bela can pick his part at random. Then the probability Jenn picks the same part is $\frac{1}{312}$, since there are a total of 312 parts. Hence the overall probability is $\frac{1}{312}$.

 # 17. Answer: (B). Note that if we find the remainder when the sum $1 + 2 + 3 + \cdots + 2020$ is divided by 2015, then we could remove this number from the sum to make it have no remainder. Note that the sums $1 + 2014$, $2 + 2013$, $3 + 2012$, $4 + 2011$, ..., $1007 + 1008$ are all multiples of 2015. Thus we only need to find the remainder when $2015 + 2016 + 2017 + 2018 + 2019 + 2020$ is divided by 2015, which is $0 + 1 + 2 + 3 + 4 + 5 = 15$. Thus removing 15 makes the sum a multiple of 2015.

 # 18. Answer: (C). By symmetry, $A_1A_5 = A_2A_6 = A_3A_7 = A_4A_8$. Since the area of a square with diagonal d is given by $\frac{d^2}{2}$, the area of $A_1A_3A_5A_7$ is $\frac{(A_1A_5)^2}{2} = \frac{5}{8}$. Thus $A_2A_6 = A_1A_5 = \frac{\sqrt{5}}{2}$.

 # 19. Answer: (C). The largest square Bob can draw on his paper has side length equivalent to the shorter side of the rectangle; it's clear that no larger square can fit. Therefore Bob's paper has sides of length 20 and k, and its diagonal d is also an integer. Now we need to find a Pythagorean triple with a side of length 20: $15 - 20 - 25$ is the smallest such triple, but this means that the shortest side of Bob's rectangle will be 15. The next smallest triangle is a $20 - 21 - 29$, which works, so $d = 29$.

 # 20. Answer: (E). Instead, consider the following scenario: slips of paper with the letters 'T' and 'H' written on them are put into a box, representing the heads and tails of the coins. Kai randomly picks one of these slips. What is the probability he picks an unfair coin?

In this case, there are $5 + 2 \times 2 = 9$ heads in the box. Of these 9 heads, 4 of them belong to unfair coins and 5 belong to fair coins. Therefore the probability of choosing an unfair coin is $\frac{4}{9}$ and the probability of choosing a fair coin is $\frac{5}{9}$. It's clear that this situation is equivalent to the set-up of the original question.

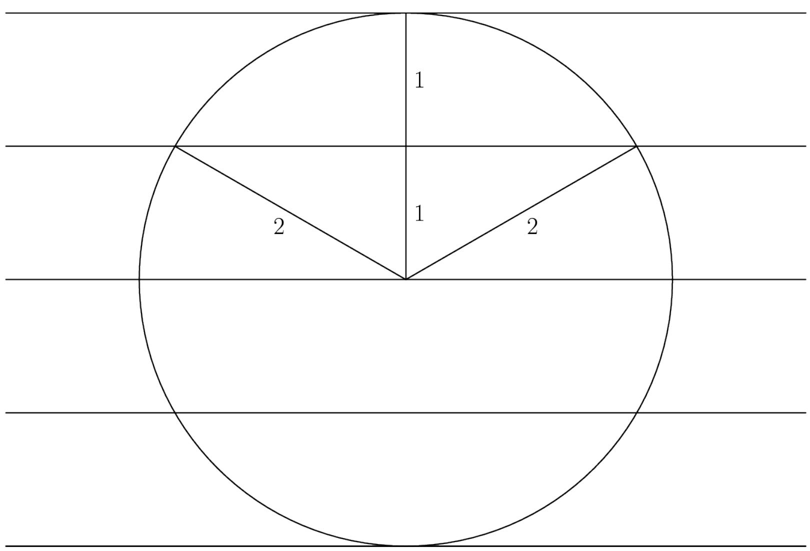
Therefore the probability that Kai gets heads again is $\frac{4}{9} \times 1 + \frac{5}{9} + \frac{1}{2} = \frac{13}{18}$.

 # 21. Answer: (B). Using $d = rt$, we get that $t = \frac{d}{r}$. Thus

$$\begin{aligned} \text{total time} &= \text{time for part 1} + \text{time for part 2} + \text{time for part 3} \\ &= \frac{30}{x} + \frac{30}{y} + \frac{30}{z} \\ &= \frac{30(yz + zx + xy)}{xyz} \\ &= \frac{30(14.1)}{9} \\ &= 47. \end{aligned}$$

 # 22. Answer: (E). The diameter of the circle is 4 (the distance between the two tangents). Therefore the radius of the circle is 2. Now drawing the radii to the bottom of the top segment, we see that we form two $30 - 60 - 90$ triangles, so the area of the top region is the area of the sector minus the area of the traingles, which is

$$\frac{1}{3} \cdot \pi \cdot 2^2 - 2 \times \frac{1}{2} \cdot 1 \cdot \sqrt{3} = \frac{4}{3}\pi - \sqrt{3}.$$



 # 23. Answer: (B). Rewrite the expression as $n^n - 2n^{n-1} + n^{n-2}$. Factoring out n^{n-2} gives

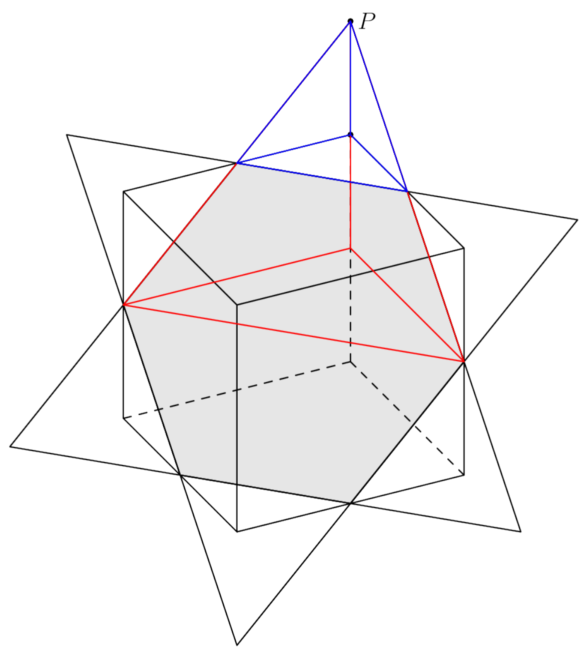
$$n^{n-2}\left(n^2 - 2n + 1\right) = n^{n-2}(n-1)^2.$$

If both n and $n - 1$ are prime, then this expression has $(n - 1) \times 3 = 3n - 3$ factors. Otherwise, we can split the expression into its prime factors, so it will have strictly more than $3n - 3$ factors. Hence both n and $n - 1$ need to be prime. But one of these numbers is even, hence $n = 2$ or $n - 1 = 2$. In the first case $2 - 1 = 1$ is not prime, but in the second case $2 + 1 = 3$ is prime. Hence there is only one integer satisfying the condition, 3.

 # 24. Answer: (C). Since $5x$ is a multiple of 30, x is a multiple of 6. n is also a multiple of 7, so the prime factors of x are 2, 3, and 7. Note that if x has any more prime factors, then the number of factors of x has at least four prime factors, but x has only 30 factors.

Now we have $x = 2^a 3^b 7^c$, with $(a+1)(b+1)(c+1) = 30$. Since $30 = 2 \times 3 \times 5$, and $a+1, b+1, c+1$ are all greater than 1, there are 6 ways to pick their values, which correspond to the 6 ways to arrange the numbers $2, 3, 5$. Therefore there are 6 such x.

 # 25. Answer: (E). Extend the edges of the hexagon, as shown. We claim that this star is the cross-section formed. Consider one of the vertices of this star, as shown:



The red and blue tetrahedra are similar and share point P. (This can be seen by noticing that the equilateral triangle faces are similar and share point P.) Hence point P lies on the (extended) edge of the cube, so it will be on the edge of Maylin's new shape.

We can repeat this for all the vertices to show that this is indeed the desired cross section.

Now we'll compute the area. The side length of the hexagon is $\frac{\sqrt{2}}{2}$, so its area is $\frac{3\sqrt{3}}{4}$.

The area of the six triangles in the star is equal to the area of the hexagon (either by just noticing they have the same side length as the hexagon or using 2007 AMC 8, problem 12). Therefore the area of $\mathcal{R}$ is $\frac{3\sqrt{3}}{2}$, so $\mathcal{R}^2 = \frac{27}{4}$.